Dynamics of Non-Equilibrium Variables:
Multiscale-multiphysics applications of fluctuating hydrodynamics
Zaragoza, Spain -- September 15-19, 2025

Fluctuating Hydrodynamics and Turbulence

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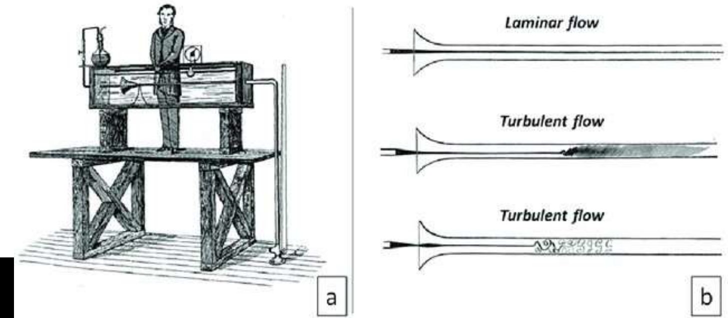
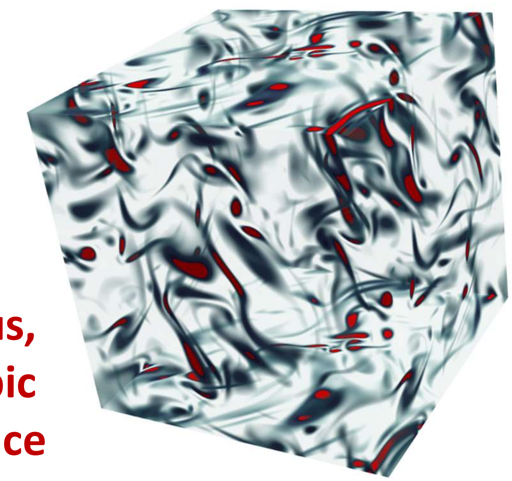
Turbulence in Fluids

"Turbulence is the most important unsolved problem of classical physics.“
Feynman Lectures (1964)





Homogenous, isotropic turbulence

Turbulence Terminology

Reynolds number $$\mathrm{Re} = \frac{\rho\, u\, L}{\eta}$$

ρ – density, η – viscosity
u, L – characteristic velocity, length scale

Taylor microscale, l_λ, is length scale at which viscosity starts to affect the dynamics of turbulent eddies in the flow. Marks the start of the "near dissipation range."

Taylor-Reynolds number, Re_λ, takes $L = l_\lambda$ and $u = \sqrt{|v^2|}$ (r.m.s. speed in eddies). Fully developed turbulence for $\mathrm{Re}_\lambda \gtrsim 100$.

Kolmogorov microscale, l_η is length scale at which viscosity dominates and turbulent energy is dissipated into thermal energy.
Marks the start of the "far dissipation range."

Compressible Navier-Stokes Equations with Forcing

Compressible Navier-Stokes equations

External forcing imposed at length scales above l_F, which marks the start of the "inertial sub-range."

continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

momentum

long-wavelength turbulent forcing

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \nabla \cdot (\rho \boldsymbol{v} \otimes \boldsymbol{v} + p\boldsymbol{I} + \boldsymbol{\Pi}) + \rho \boldsymbol{a}_T = 0$$

$\boldsymbol{\Pi}$ – Stress tensor

energy

energy "thermostat"

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\boldsymbol{v}\,(\rho E + p) + \boldsymbol{Q} + \boldsymbol{\Pi} \cdot \boldsymbol{v}) + [\rho \boldsymbol{a}_T \cdot \boldsymbol{v} - \langle \rho \boldsymbol{a}_T \cdot \boldsymbol{v} \rangle] = 0$$

$\mathbf{Q}$ – Heat flux

Optional: assume incompressibility and uniform isothermal conditions

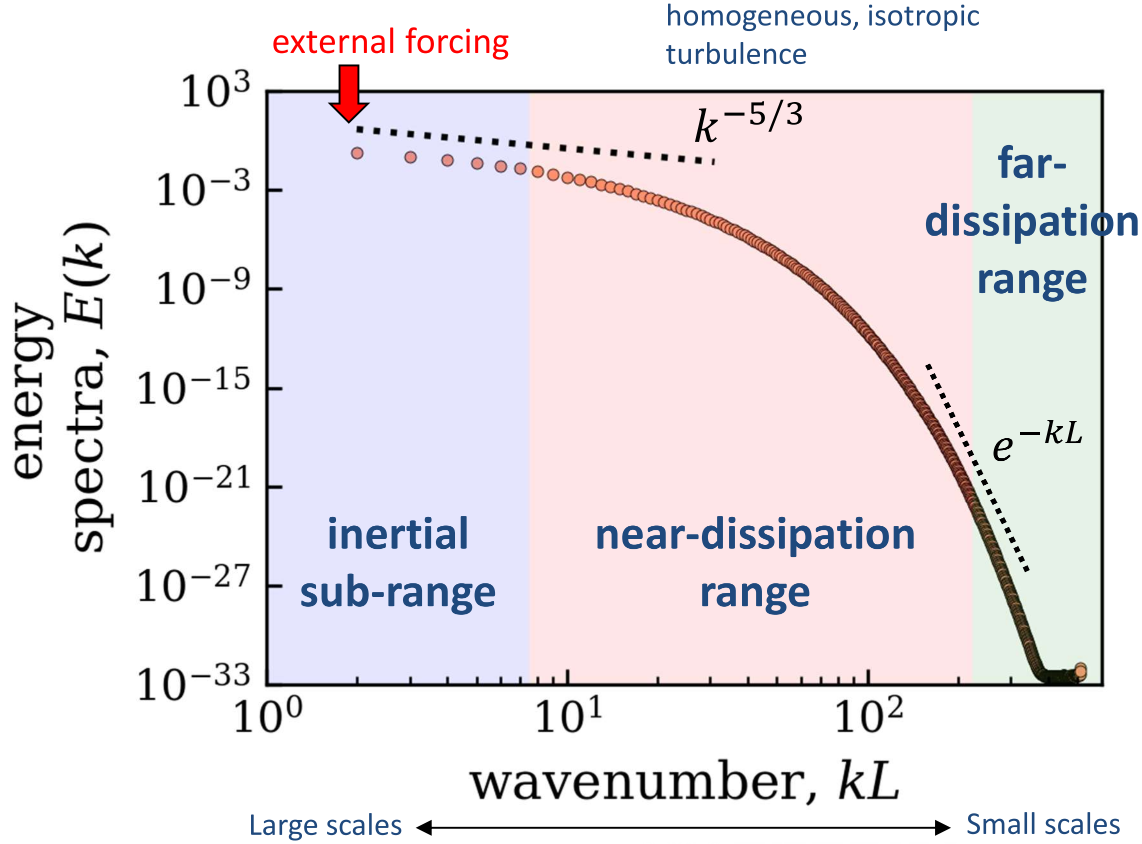
Standard Picture of Turbulence: Energy Spectrum

inertial sub-range
- inviscid energy cascades from large to small eddies
- $k^{-5/3}$ Kolmogorov scaling

near-dissipation range
- viscous effects start appearing
- intermittency starts growing

far-dissipation range
- viscous dissipation dominates
- strong intermittency

Numerical simulation of homogeneous, isotropic turbulence



Thermal Fluctuations and Turbulence: Standard View

At standard conditions there are $N \approx 10^4$ air molecules per cubic mean free path so von Neumann (1949) argued that thermal fluctuations becomes relevant in turbulent flows only for length scales of order the mean free path.

Moser (2006) derived this ratio of Kolmogorov length scale, l_η, to mean free path, l_{mfp},

$$\frac{l_\eta}{l_{\text{mfp}}} \cong \frac{1}{4} \frac{\sqrt{\text{Re}_\lambda}}{\text{Ma}}$$

Ma – Mach number

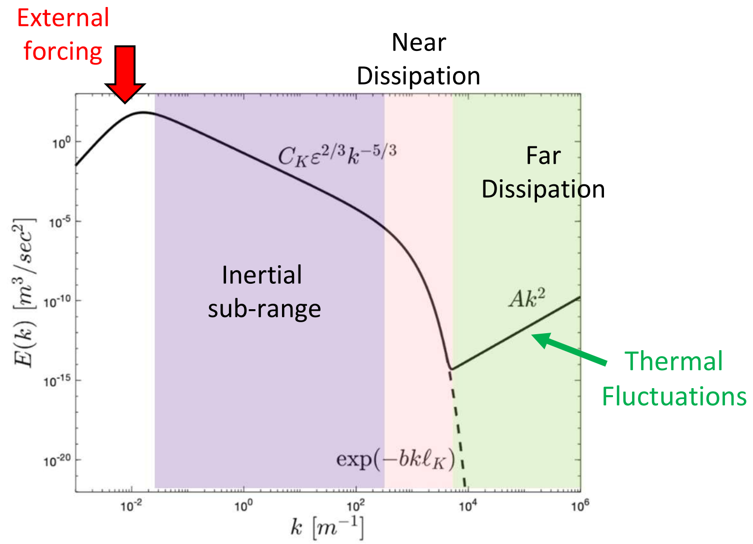
For typical subsonic turbulence, $l_\eta \approx 10\, l_{\text{mfp}} - 100\, l_{\text{mfp}}$, so thermal fluctuations were only expected to be noticeable *deep* into the far dissipation range.

Thermal Fluctuations and Turbulence: Dissenting Opinion

Betchov (1957) had a dissenting opinion and argued that thermal fluctuations would be noticeable in turbulent flows at scales larger than a mean free path.

Bandak et al. (2022) estimated the length scale at which the exponentially decaying energy density of turbulence would equal the energy density of thermal fluctuations.

By their estimates the thermal fluctuation spectrum, which goes as k^2, dominates somewhere close to the border between the near and far dissipation regions.



Bandak et al., *PRE*, 2022

So what? Who cares?

In laboratory experiments, the Taylor microscale is $l_\lambda \approx 1 - 10$ mm and the Kolmogorov microscale is $l_\eta \approx 1 - 10$ μm.

The current generation of turbulence experiments cannot resolve the Kolmogorov microscale but the next generation is being designed to see it.

There are *many* theories and simulations for the far dissipation regime but they are all based on the deterministic Navier-Stokes equations.

But if thermal fluctuations dominate turbulence in the far dissipation regime then what's the point of those experiments, theories, and simulations?

Compressible Fluctuating Hydrodynamics

Stochastic Navier-Stokes equations, as formulated by Landau and Lifshitz.

continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

momentum

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \nabla \cdot \left(\rho \boldsymbol{v} \otimes \boldsymbol{v} + p\boldsymbol{I} + (\boldsymbol{\Pi} + \widetilde{\boldsymbol{\Pi}})\right) + \rho \boldsymbol{a}_T = 0$$

energy

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot \left(\boldsymbol{v}\,(\rho E + p) + (\boldsymbol{Q} + \widetilde{\boldsymbol{Q}}) + (\boldsymbol{\Pi} + \widetilde{\boldsymbol{\Pi}}) \cdot \boldsymbol{v}\right) + [\rho \boldsymbol{a}_T \cdot \boldsymbol{v} - \langle \rho \boldsymbol{a}_T \cdot \boldsymbol{v} \rangle] = 0$$

stress tensor

heat flux

stress tensor

Add a Langevin-type white-noise **stochastic flux** to Navier-Stokes eqns., while satisfying **fluctuation-dissipation balance**

Dissipative and Stochastic Fluxes

dissipative stress tensor (deterministic)

$$\Pi_{ij} = -\eta \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \delta_{ij} \left[\left(\lambda - \frac{2}{3}\eta \right) \nabla \cdot \vec{v} \right]$$

η - shear viscosity

λ - bulk viscosity

fluctuating stress tensor (stochastic) [Español, *Physica A* (1998)]

$$\widetilde{\Pi}(\mathbf{r}, t) = \sqrt{2k_B T \eta}\, \widetilde{\mathcal{Z}}^{\Pi} + \left(\pm \sqrt{\frac{k_B \lambda T}{3}} - \frac{\sqrt{2k_B \eta T}}{3} \right) \mathrm{Tr}(\widetilde{\mathcal{Z}}^{\Pi}) \mathbb{I}$$

$$\widetilde{\mathcal{Z}}^{\Pi}_{3\times 3} = \frac{\mathcal{Z}_{3\times 3} + \mathcal{Z}^{T}_{3\times 3}}{\sqrt{2}}$$

dissipative heat flux (deterministic)

$$\vec{Q} = -\kappa \nabla T$$

κ – thermal diffusivity

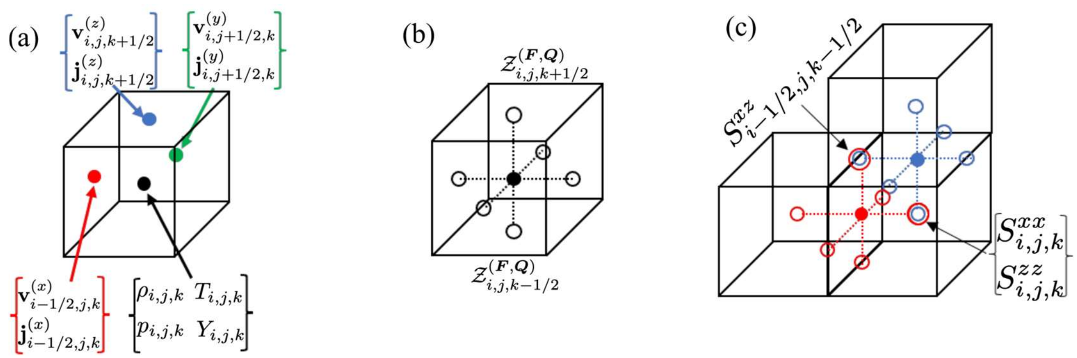
fluctuating heat flux (stochastic)

$$\widetilde{\vec{\mathbf{Q}}} = \sqrt{2\kappa k_B T^2}\ \mathcal{Z}^{Q}$$

Finite-Volume Numerical Scheme

During Aleks Donev's post-doc at Berkeley Lab we developed several efficient, accurate numerical schemes for fluctuating hydrodynamic equations.
We've been refining these stochastic PDE schemes for over a decade.

staggered grid finite-volume discretization

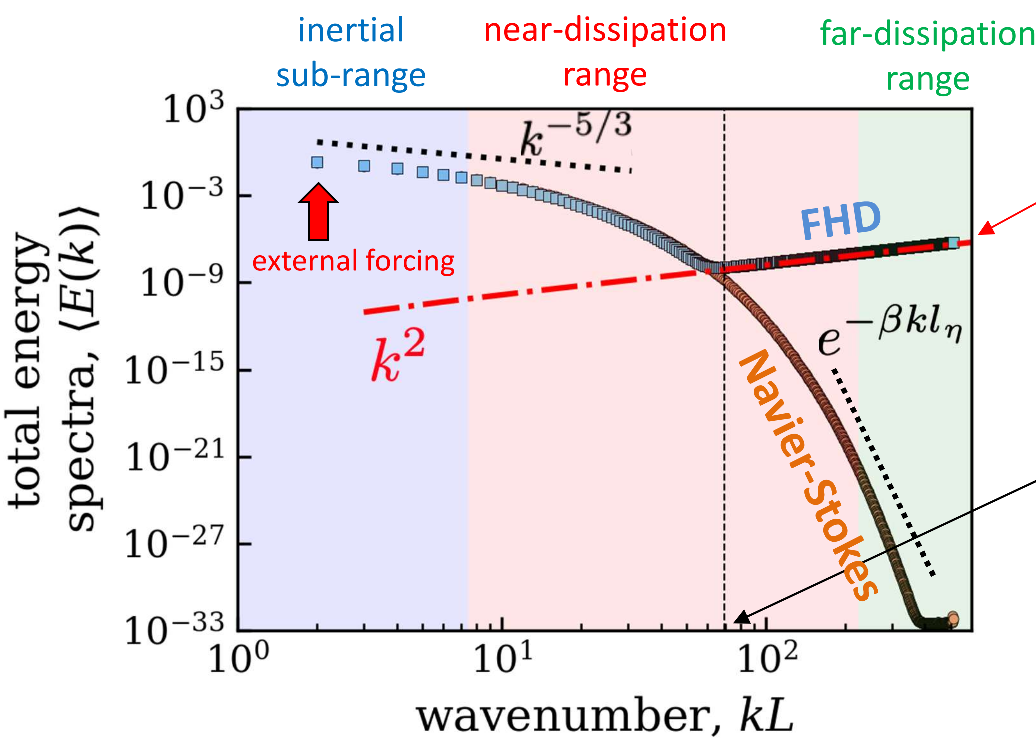


optimized 3-stage Runge-Kutta explicit integration

cell-centered (scalars):
density, energy, temperature
face-centered (vectors):
velocity and heat fluxes
edge- and cell-centered (tensors):
stress tensor

Srivastava et al., *Staggered scheme for the compressible fluctuating hydrodynamics of multispecies fluid mixtures*, Phys. Rev. E, 107 (2023)

Fluctuating Hydrodynamics of Turbulence: Energy Spectrum



$\mathrm{Re}_\lambda = 52$ $\mathrm{Ma} = 0.2$

thermal spectra (equipartition)

$$E_{\mathrm{th}}(k) = \frac{3k_B\langle T \rangle}{2\langle \rho \rangle} 4\pi k^2$$

thermal crossover length

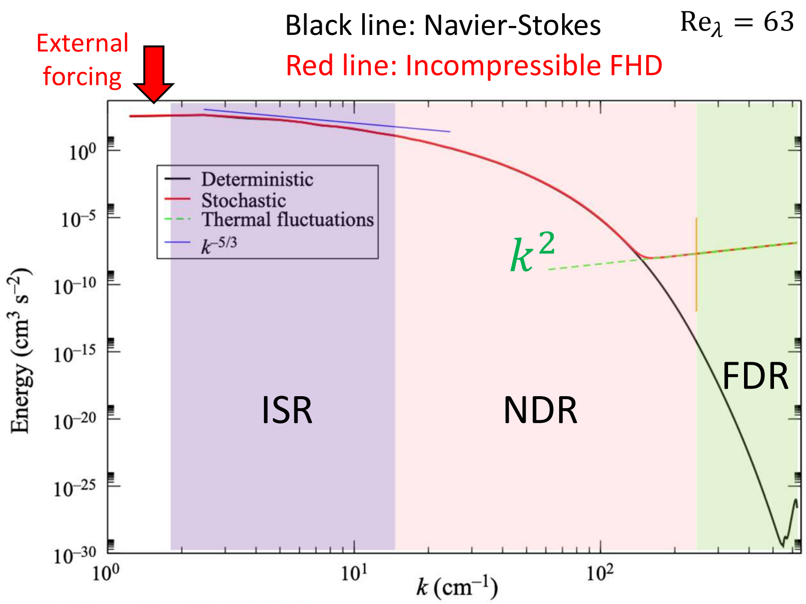
$$u_\eta^2 l_\eta \exp\left(-\beta k_{\mathrm{th}} l_\eta\right) \approx \frac{k_B \langle T \rangle}{\langle \rho \rangle} k_{\mathrm{th}}^2$$

We find that thermal fluctuations dominate turbulence at length scales significantly **larger** than the Kolmogorov microscale.

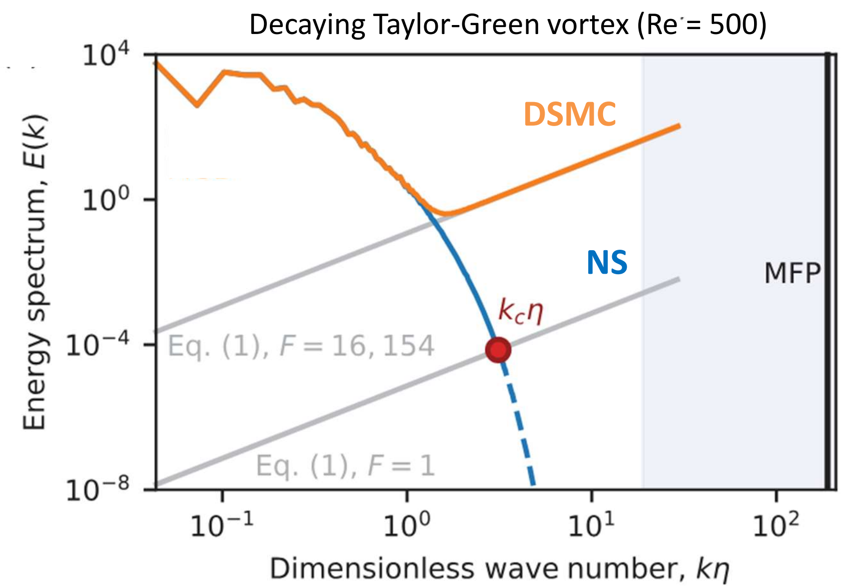
Aleks' comment

Independent Verifications of Energy Spectrum

Our turbulence simulations using incompressible fluctuating hydrodynamic and particle-based simulations (DSMC) by the Sandia group both indicate that the crossover where thermal fluctuations dominate turbulence lies within the Near Dissipation Regime (NDR).



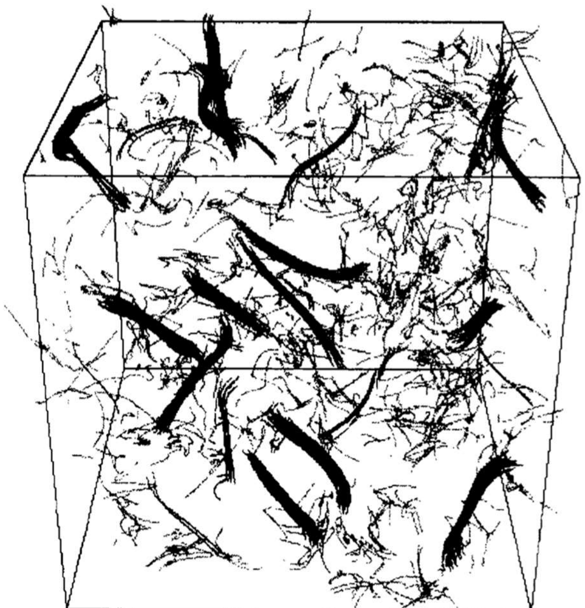
Bell et al., *JFM*, 2022



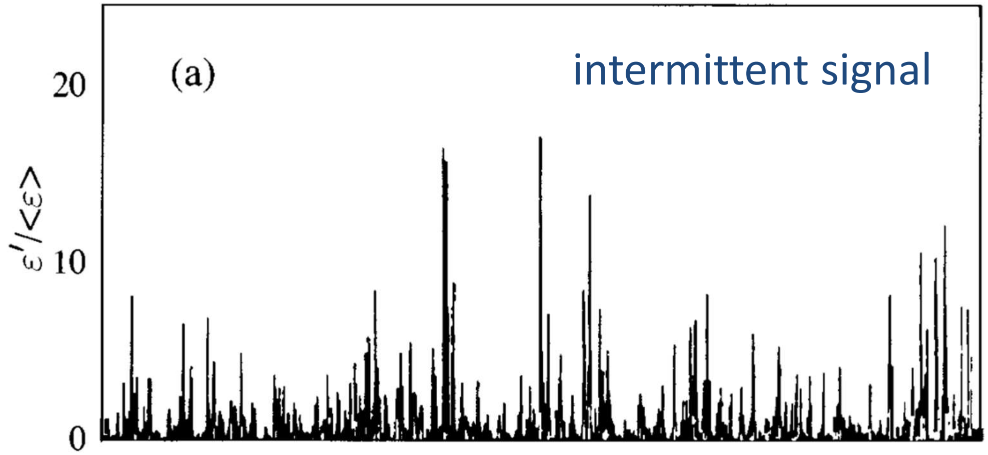
McMullen et al., *PRL*, 2022

Standard Picture of Turbulence: Intermittency

intermittent vortex filaments



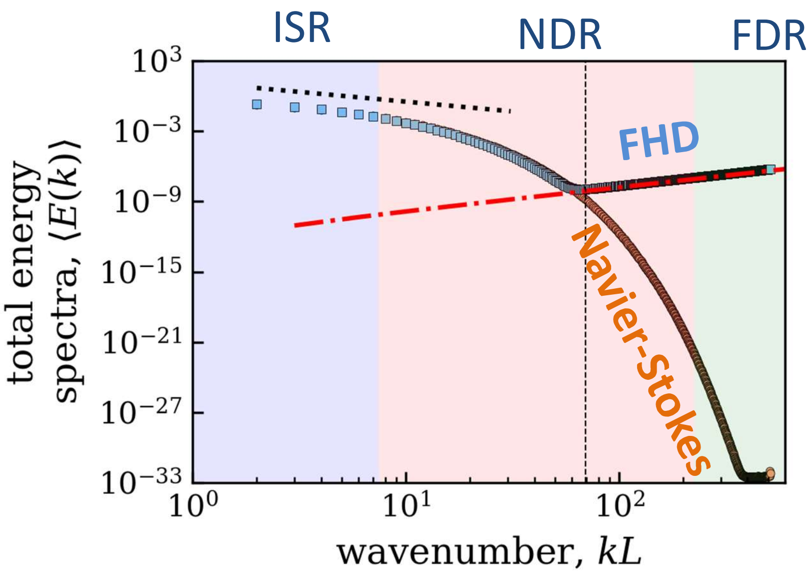
She et al., *Nature*, 1990

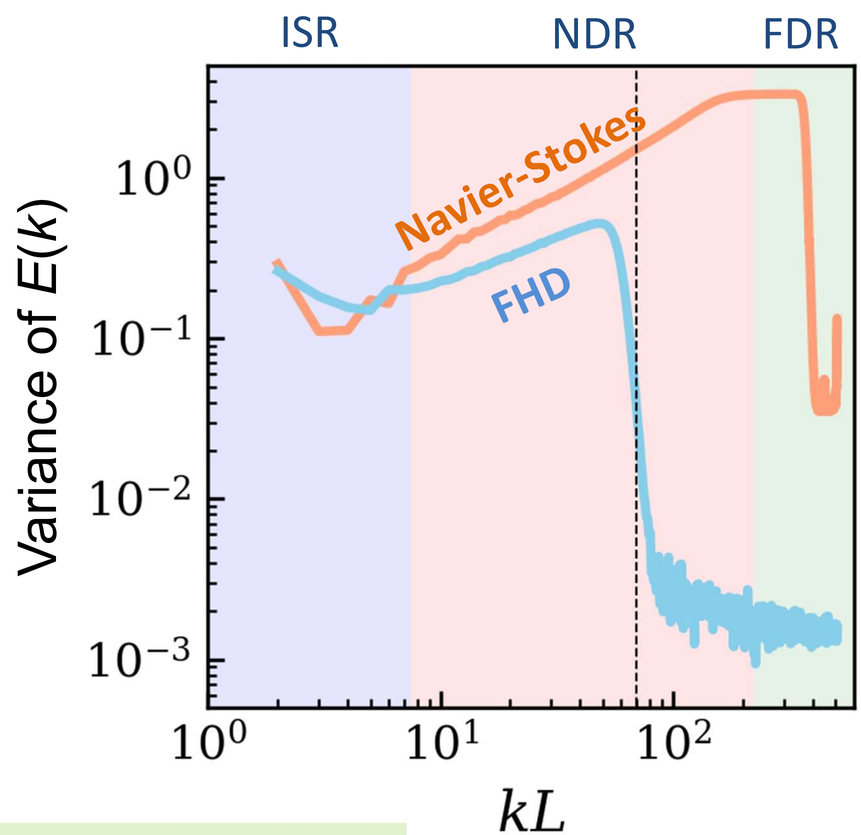


Frisch, *Turbulence: legacy of Kolmogorov*, 1995

turbulence intermittency
- highly non-Gaussian hydrodynamic fields
- localized bursts of extreme activity in a sea of largely quiet flow

Reduced Temporal Intermittency in the Dissipation Range



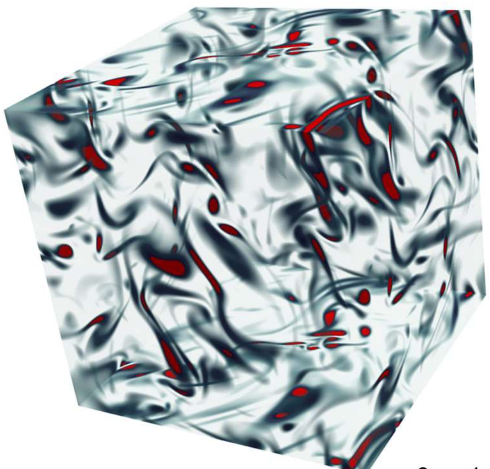


Temporal fluctuations in the kinetic energy are substantially reduced across the NDR and FDR

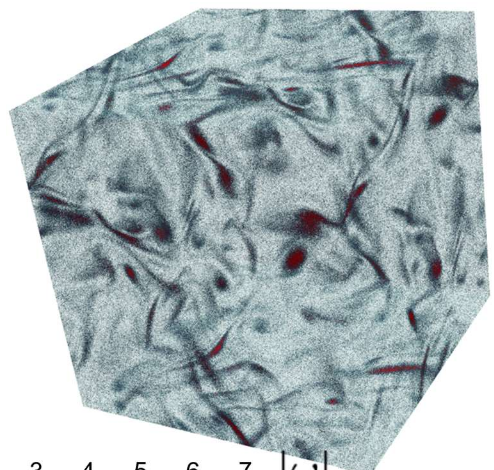
Srivastava et al., *Molecular fluctuations inhibit intermittency in compressible turbulence, arXiv:2501.06396* (2025)

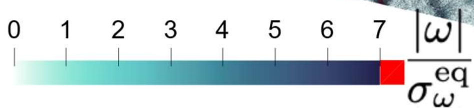
Reduced Intermittency in the Dissipation Range: Vorticity

Navier-Stokes

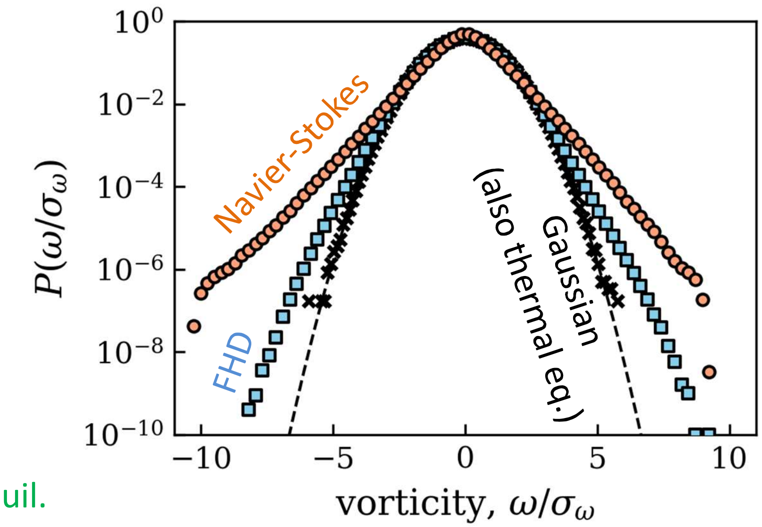


FHD

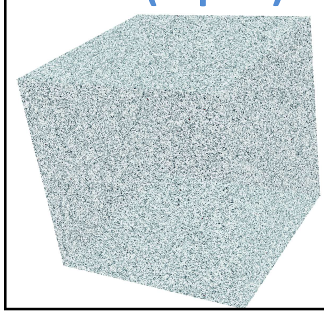




local vorticity distribution



FHD (equil.)



Sharp vorticity bursts homogenized by thermal fluctuations

0.2mm^3 of nitrogen gas, discretized over 1024^3 finite-volume grid
2048 AMD MI250X GPUs; 12 hours for 1ms of simulation

Concluding Remarks

- Molecular fluctuations fundamentally modify compressible turbulence at large and practically-relevant length scales >> molecular mean free path
- Our results indicate that intermittency effects are substantially reduced across the entire dissipation range of turbulence
- Fluctuating hydrodynamics is a (perhaps the only) tractable numerical method to model these effects at such large length scales
- We have developed a robust numerical framework for modeling compressible FHD & turbulence

Thank you for staying for my talk!

https://github.com/AMReX-FHD/FHDeX

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