Homework 9 (Due Tuesday, February 28th)

1. Consider an ideal gas of N indistinguishable, independent particles in the extremely relativistic limit in which the energy of a particle is $\epsilon = cp = c|\mathbf{p}|$.

(a) Show that the classical canonical partition function is

$$Q_N = \frac{1}{N!}\left[8\pi V\left(\frac{kT}{hc}\right)^3\right]^N$$

(b) Find the classical canonical partition function for a similar extremely relativistic gas of N indistinguishable, independent particles but in the case where they are constrained to move in a one-dimensional system of length L.

(c) Find the equation of state, $P(L,T)$, for the one-dimensional extremely relativistic gas of N indistinguishable, independent particles.

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2. Consider a monatomic ideal gas of spin 1/2 atoms in a uniform magnetic field B. In addition to the usual kinetic energy, each atom has an energy of $\pm\mu B$ where μ is the atom's magnetic moment. The gas is so dilute that the interatomic forces may be neglected (i.e., the gas is ideal).

(a) Show that the classical canonical partition function is

$$Q_N = \frac{1}{N!}\left(\frac{2V}{\lambda^3}\cosh(\beta\mu B)\right)^N$$

(b) Find the internal energy of this gas.

(c) Find the heat capacity C_V of this gas.

(d) Accurately graph C_V/Nk versus $kT/\mu B$ using a semi-log scale (linear in the vertical; logarithmic in the horizontal, going from 0.1 to 10). Explain why for very low and very high temperatures the heat capacity per particle is $3k/2$.

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3. The energy levels of a quantum-mechanical, one-dimensional anharmonic oscillator may be approximated as,

$$\epsilon(n) = (n+\tfrac{1}{2})\hbar\omega\left\{1-\alpha(n+\tfrac{1}{2})\right\}$$

where $n = 0, 1, 2, \ldots$ and α is a constant.

(a) Show that the canonical partition function for a single anharmonic oscillator is approximately, in the limit $|\alpha| \ll 1$,

$$Q_1 = f(x) + \alpha x\frac{d^2 f}{dx^2}$$

where $f(x) = \frac{1}{2}\mathrm{csch}(x/2)$ and $x = \hbar\omega/kT$.

(b) From the result in part (a) show that the specific heat for a system of N anharmonic oscillators is approximately,

$$C \approx Nk\left[1 + a\,\frac{\alpha kT}{\hbar\omega}\right]$$

in the high temperature limit ($kT \gg \hbar\omega$) and find the numerical constant, a.