Homework 10 (Due Thursday, March 8th)

1. (a) From the probabilities in the grand canonical ensemble,

$$\mathcal{P}_i = \frac{e^{-\beta E_i + \beta\mu N_i}}{\sum_i e^{-\beta E_i + \beta\mu N_i}}$$

show that

$$U = \sum_i E_i \mathcal{P}_i = -\left(\frac{\partial}{\partial\beta}\right)_z \ln \mathcal{Q}$$

and that

$$S = -k \sum_i \mathcal{P}_i \ln \mathcal{P}_i = k \ln \mathcal{Q} + k\beta U - k\beta\mu N$$

(b) The grand canonical partition function for an ideal gas in the classical limit is

$$\mathcal{Q} = \exp\left(\frac{zV}{\lambda^3}\right)$$

where $z = e^{\beta\mu}$ and $\lambda = h/\sqrt{2\pi mkT}$. Using $\mathcal{Q}$ find $U(T,V,N)$, $P(T,V,N)$, and $S(T,V,N)$; you may use any of the results derived in part (a) or in Section 4.3 of Pathria and Beale.

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2. The Fermi energy is defined as

$$\epsilon_F = \frac{\hbar^2}{2m}\left(\frac{6\pi^2 N}{gV}\right)^{2/3}$$

where g is the degeneracy factor; $g = 2s+1$ for a particle of spin s. Estimate the Fermi energy (in electron volts) and the Fermi temperature, $T_F = \epsilon_F/k$ for:

(a) Electrons in solid copper;

(b) Protons in the nucleus of an atom;

(c) He^3 atoms in the liquid phase (atomic volume is $4.62 \times 10^{-29}\mathrm{m}^3$ per atom);

(d) Electrons in a white dwarf star that is composed of helium with a stellar density of $10^{10}\mathrm{kg/m}^3$ and a temperature of 10^7 K.

In each case assume the particles can be treated as non-relativistic but compare the Fermi energy with the relativistic energy $\epsilon_R \equiv mc^2$ and comment on whether the non-relativistic assumption is justified. Note that you will need to look up physical parameters, such as the density of copper and the mass of an electron.