Can a system of elastic hard spheres mimic the transport properties of a granular gas?

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The simplest model of a granular fluid in the rapid flow regime consists of a gas of (smooth) inelastic hard spheres (IHS) characterized by a constant coefficient of normal restitution α. The inelasticity of collisions produces a decrease of the mean kinetic energy (or granular temperature) with a cooling rate $\zeta \propto 1-\alpha^2$. The same cooling effect can be generated in a gas of *elastic* hard spheres (EHS) by the application of an effective drag force with a friction coefficient $\frac{1}{2}\zeta$. At a macroscopic level of description, the hydrodynamic balance equations of mass, momentum, and energy for the IHS gas are (formally) identical to those for the frictional EHS gas. However, the microscopic dynamics is physically quite different in both systems: in the IHS system the particles move freely between two successive collisions but they lose energy after each collision; in the EHS case, on the other hand, energy is conserved by collisions but the particles lose energy between collisions due to the action of the effective drag force. During a certain small time step, only the small fraction of colliding particles are responsible for the cooling of the system in the IHS case, whereas all the particles contribute to the cooling in the EHS case. Therefore, there is no reason in principle to expect that the relevant physical properties (e.g. the velocity distribution function) are similar for IHS and frictional EHS under the same conditions.

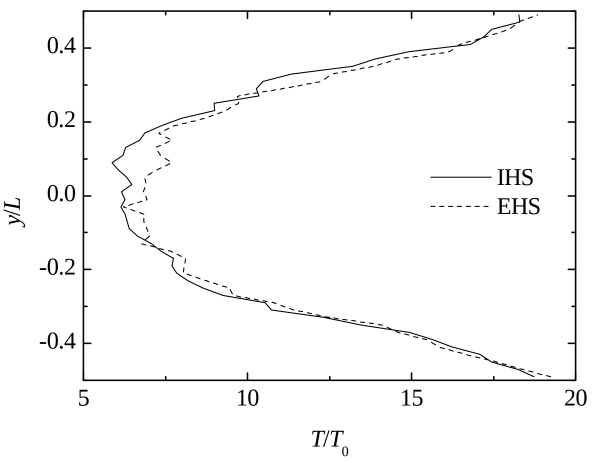

FIG. 1: Temperature profile at $t=\tau_0$.

For instance, in the so-called homogeneous cooling state the solutions to the respective Boltzmann equations for IHS and EHS differ. While the distribution function is a (time-dependent) Gaussian for EHS, deviations from a Gaussian (as exemplified by a nonzero kurtosis and by an overpopulated high energy tail) are present in the case of IHS. A similar situation occurs in the homogeneous nonequilibrium steady state driven by a white noise forcing. Notwithstanding this, the differences between the homogeneous solutions for IHS and EHS are not quantitatively important and so it is still possible that both systems exhibit comparable departures from equilibrium in inhomogeneous states where transport of momentum and/or energy is the relevant phenomenon. The investigation of this possibility is the main aim of this work.

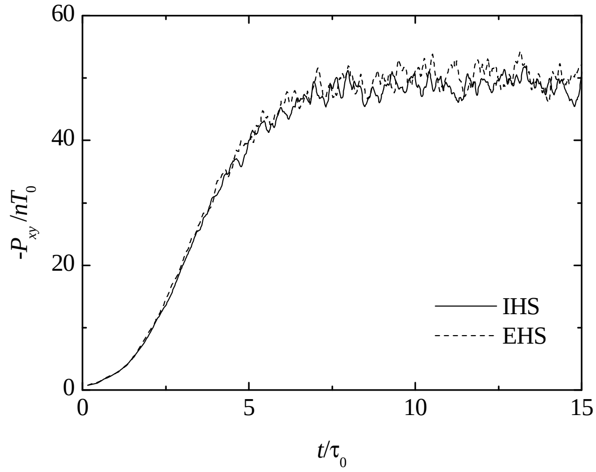

FIG. 2: Time evolution of the shear stress.

On the theoretical side, we have compared the transport coefficients obtained from the Chapman–Enskog method for IHS and the equivalent gas of EHS. The results show that the optimal choice for the diameter of the elastic spheres is smaller than the actual diameter of the inelastic spheres. Next, we have carried out DSMC simulations in both systems for the simple shear flow problem. In general, it is found that the equivalent EHS system succeeds in capturing the main nonequilibrium transport properties of the underlying IHS system, as illustrated in Figs. 1 and 2 for a coefficient of restitution $\alpha=0.9$, a shear rate $a=4\tau_0^{-1}$ (where τ_0 is the initial mean free time of the IHS gas), and a separation $L=2.5\lambda$ (where λ is the mean free path of the IHS gas) between the moving plates. Figure 1 shows the temperature profile $T(y)/T_0$ (where T_0 is the initial granular temperature) at $t=\tau_0$ starting from an *equilibrium* distribution function, while Fig. 2 shows the time evolution of the reduced shear stress $-P_{xy}(t)/nT_0$ (where n is the number density) starting from a *local equilibrium* distribution function.

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