July 2009

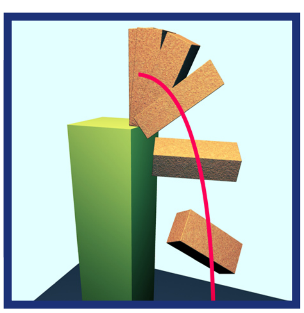
www.AnimationPhysics.com

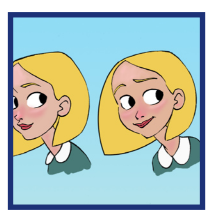
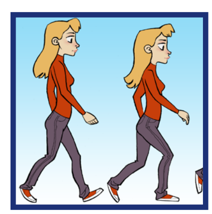
One of the major problems for the inbetweeners is that it is much more difficult to make a drawing on an arc…Drawings made as straight inbetweens completely kill the essence of the action.

from The Illusion of Life, F. Thomas & O. Johnston

Physics of Paths of Action

This tutorial explains basic paths of action, such as the flight of a thrown ball or a brick tipping off the side of a table. In other words, how things move when they're not moving in a straight line.

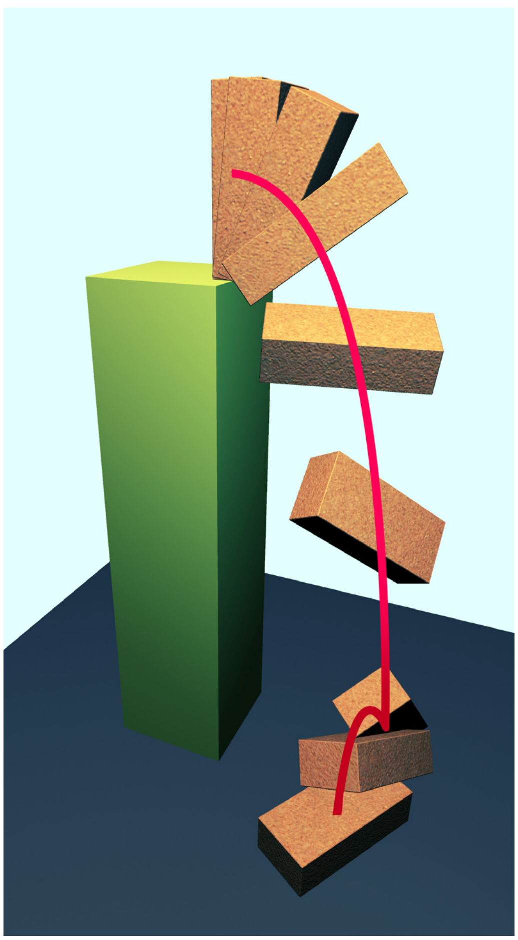


Brick Drop Animation Test

► The brick drop is a basic animation test that introduces several principles of animation. We'll use it as the unifying example in this tutorial.

In this test a brick tips over from a height (say a table), falls through the air, and strikes the ground. On impact the brick may bounce or even break into pieces.

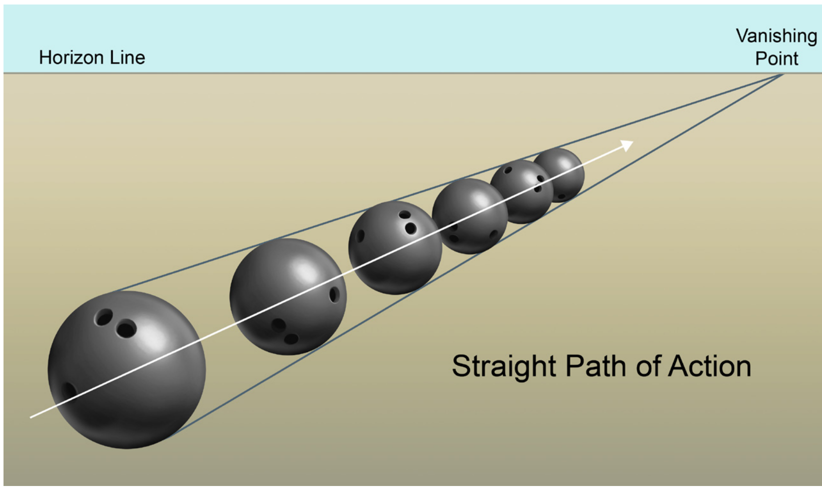


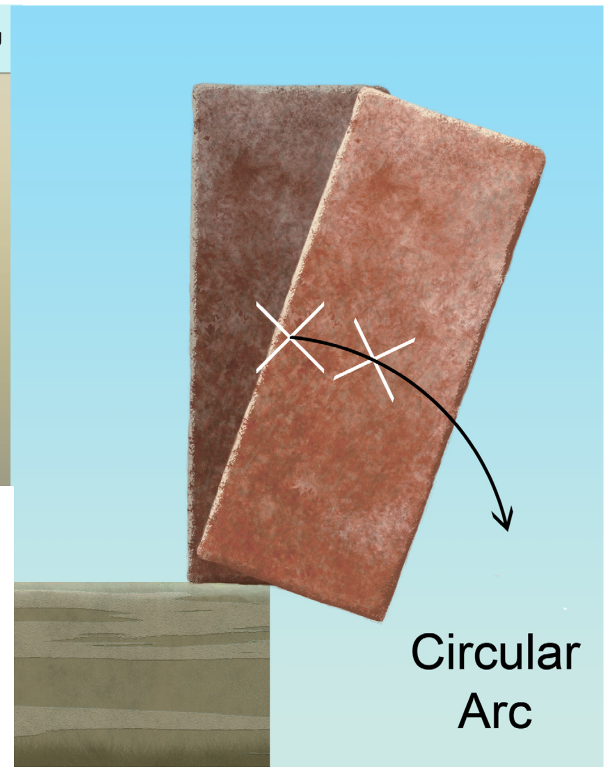
The main principle discussed in this tutorial is the path of action, such as in tipping, falling, and bouncing. But we'll also talk about other elements, such as the timing of tipping over (and how it creates anticipation) as well as the turning motion of the falling brick, which could be viewed as a secondary path of action.

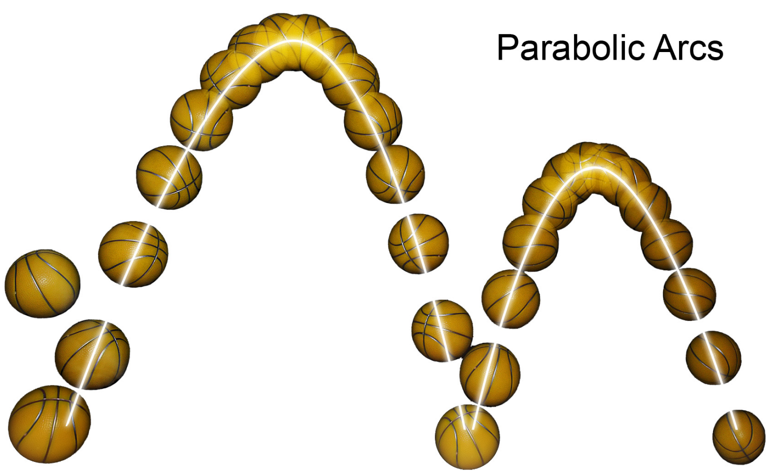
Paths of Action

► The path of action indicates the path along which an object or a character is moving through space. Physicists call this a trajectory.

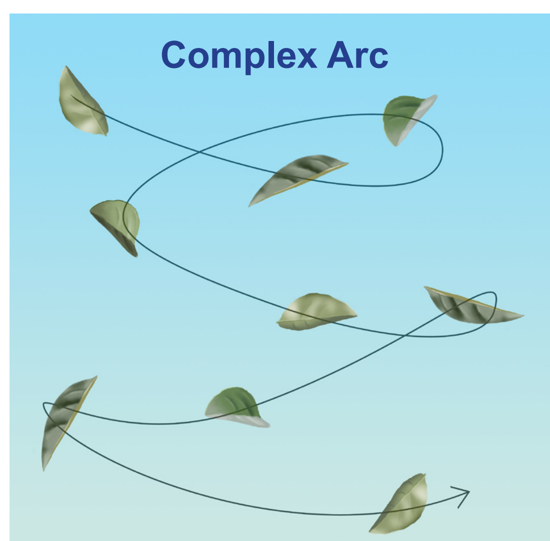
Objects rarely move in straight lines; it's much more common for them to move in arcs.







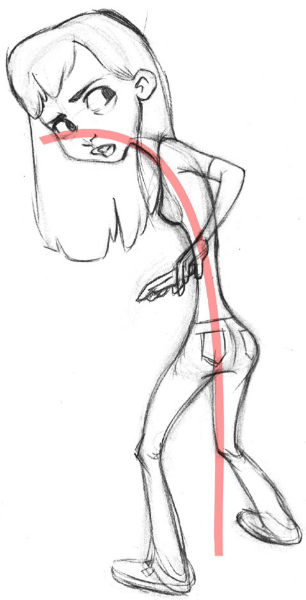
Note: **Typically, the more forces acting on an object, such as gravity, friction, etc., the more complicated the motion. Some forces, such as air resistance, vary with time, which makes the timing, spacing, and path of action even more complicated.**

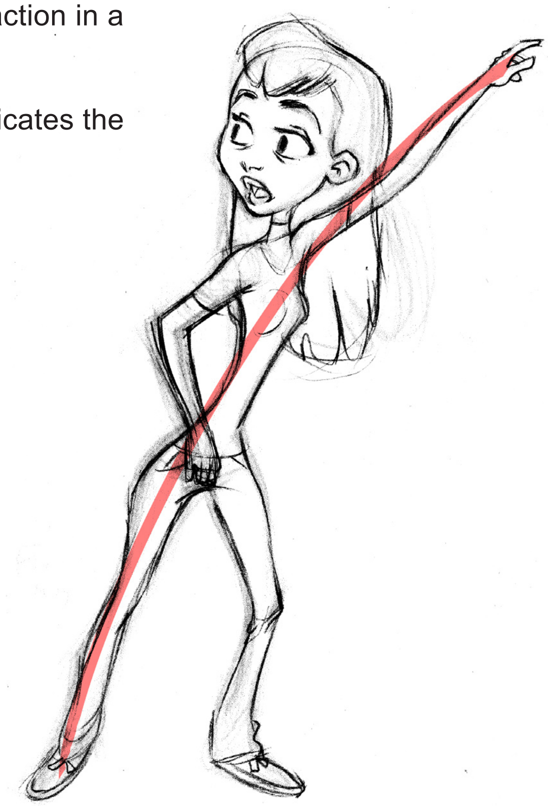


Line of Action vs. Path of Action

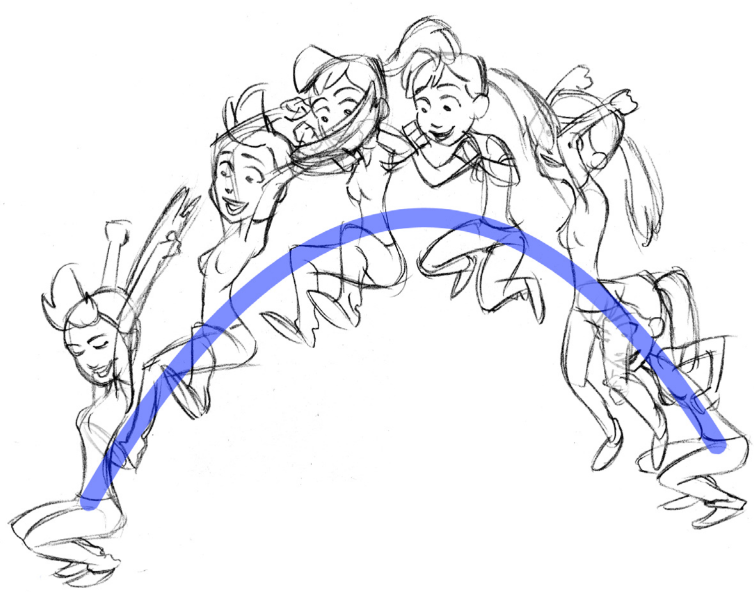
► It's important that you don't confuse the line of action in a drawing with the path of action in an animation.

Individual drawings have a line of action, which indicates the visual flow of action in that single drawing.





The path of action is the trajectory for a sequence of drawings, such as the parabolic arc in this jump.



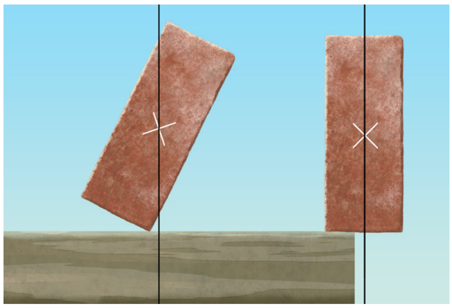
The path of action is usually associated with the primary action but we can also consider paths of action for secondary actions, such as the motion of a character's hand, arm, foot, etc.

Circular Arcs

► One of the simplest paths of action is the circular arc.

For example, a brick rotates about a point as it tips because friction tends to keep the brick from sliding until it's about to lose contact with the table.

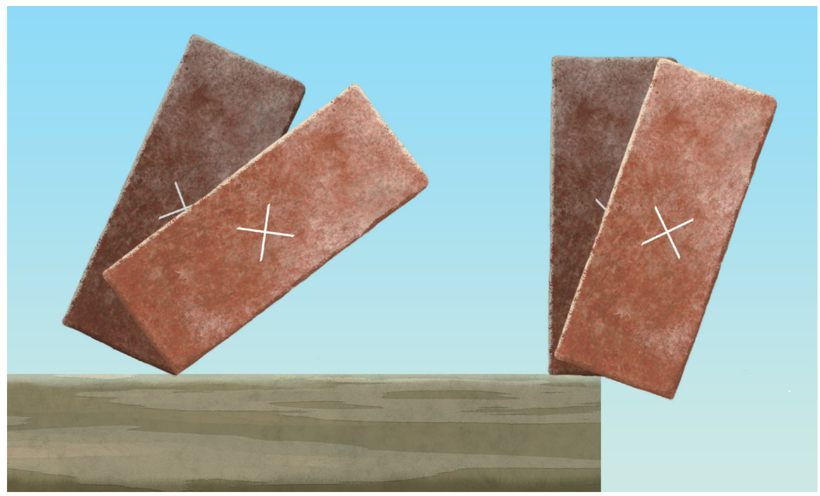
The center of the circular arc is the point about which the brick rotates.

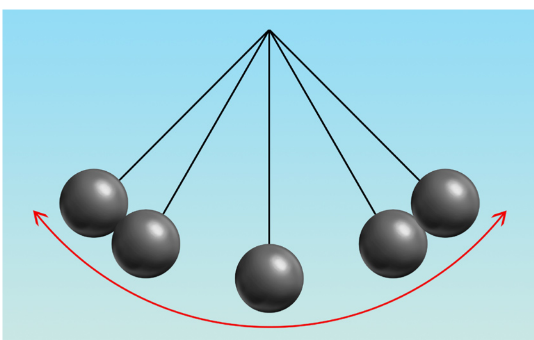


A brick may be off-balance in two ways:

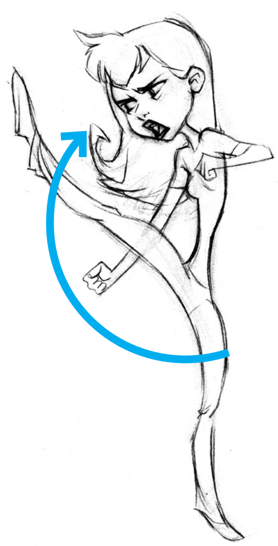
* Center tipped past point of contact

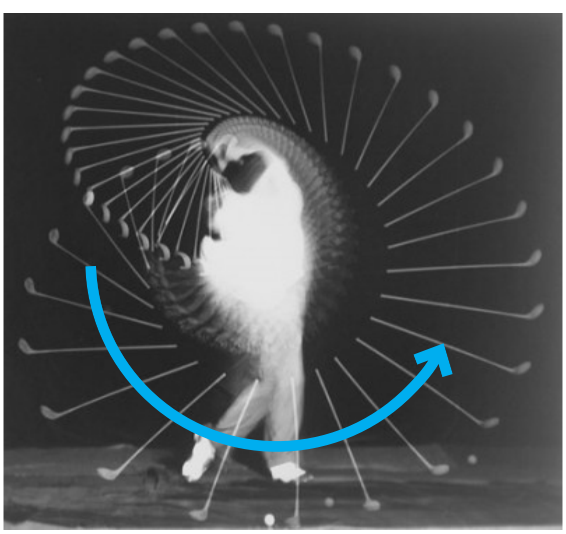
* Center past the edge of the table





Circular arcs are common because motion is often constrained by a fixed pivot point, such as a joint.





The timing of the motion on a circular arc may be uniform and even (as in the lower right part of this golf swing), or it may slow out (as in the lower left) or slow in (as in the upper half).

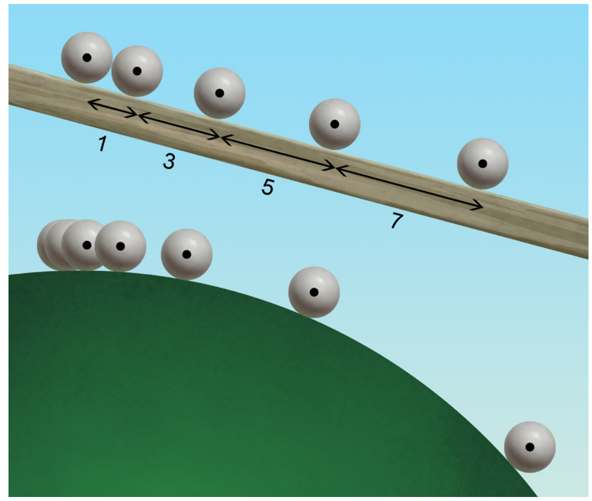
Exponential Spacing

► Exponential spacing occurs when the acceleration itself accelerates, such as when a ball rolls down a slope that is increasingly steeper and steeper.

As discussed in the tutorial "Physics of Timing and Spacing", a ball rolling down a straight ramp slows out with spacings given by the Odd Rule, that is, in the ratios of 1:3:5:7:9:...

On the other hand, if the slope is a circular arc then the spacings increase exponentially (e.g., 1:2:4:8:16:...). Slowing out from the peak of the hill is very slow initially but then gains speed rapidly as the slope steepens.

Similarly, a ball rolling up a circular hill slows in with exponential spacing, moving very slowly as it nears the apex.



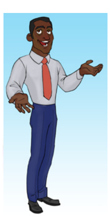
In the Odd Rule, the spacings increase by a constant added increment. In exponential spacing, the spacings increase by a constant multiplied factor, as with compound interest in banking.

Another example of exponential spacing occurs when an object tips over. The motion is very slow initially but gains speed rapidly as the angle increases.






Tipping brick at 1/24th second per frame



Important: **Anticipation is one of the principles of animation and it may be achieved in the brick drop by having the brick initially rock backwards (as an antic) before falling forward. Exponential spacing helps to create anticipation because the motion near the apex is slow, allowing the audience to cue in on the what's to occur, before the main action in which the brick slows out in a dramatic fashion.**

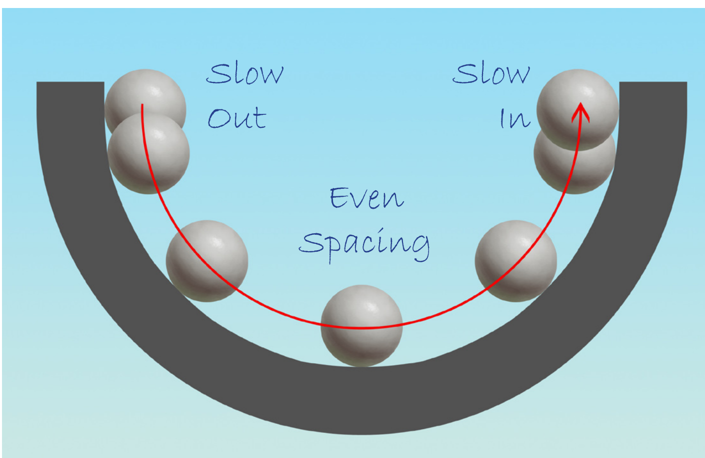
Pendulum Spacing

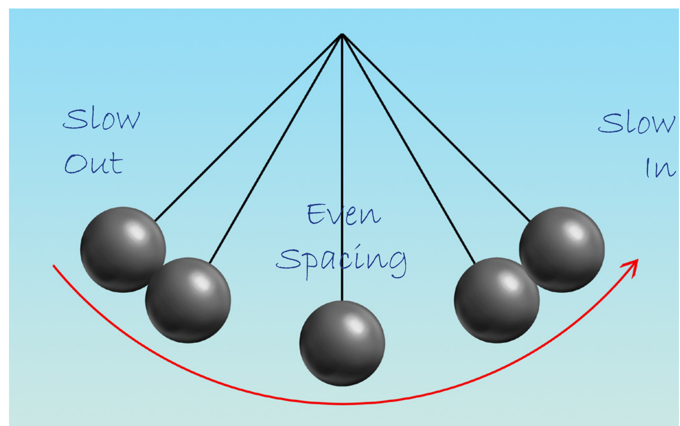
► A pendulum also swings in a circular arc but its spacing is not exponential.

The spacing of a pendulum is best understood by viewing it as a ball rolling from side to side in a circular bowl (or a skater in a half-pipe).

At the left and right extremes the slope is steepest so the acceleration is the greatest at those turning points.

At the center the slope flattens out so while the speed is the greatest in the center, that speed is more or less constant so the spacing is nearly even.



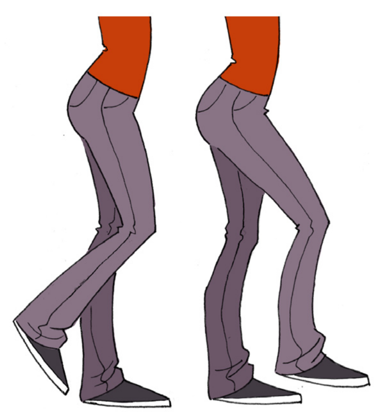


As with falling, the pendulum's timing is independent of the weight (as long as friction doesn't slow down the motion).

The swing of the leg in the passing position of a walk cycle is a good example of pendulum motion.

The motion of an arm or a leg rotating about a joint slows in and out as a pendulum if the muscles are relaxed such that the limb swings primarily by the force of gravity.

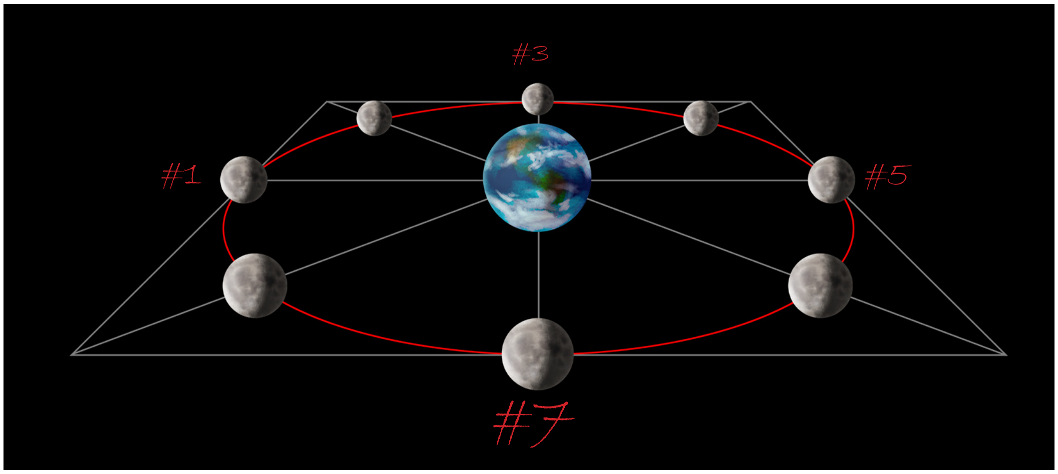
The larger the arc's radius, the longer a pendulum takes to swing back and forth. This is one reason that creatures with longer legs tend to have a slower stride (for example, a large dog, such as a Great Dane, has a lumbering walk when compared with the brisk stride of a chihuahua).

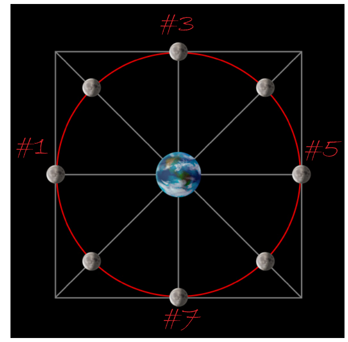


Circular Arcs in Perspective

► A circle viewed in perspective is an ellipse.

The distortion of spacing from background to foreground creates dramatic timing, even for uniform rotation.





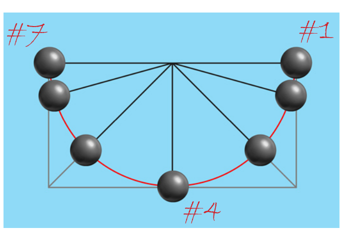
Visually, the distance travelled across the screen from pose #1 to #5 is about the same as from #6 to #8. Since the motion in the foreground occurs in half the time as the motion background, this foreground motion appears to have twice the speed. (Note: Earth-Moon orbit is not to scale)

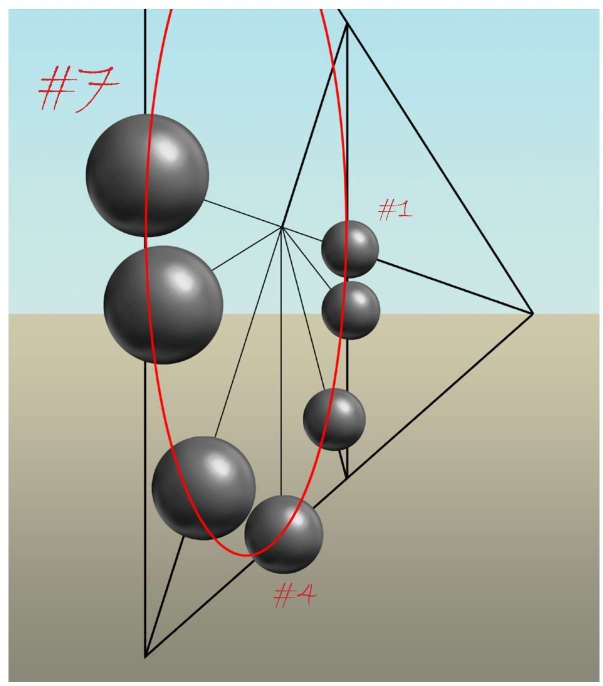
For a pendulum the effect of perspective is even more noticeable on the timing and spacing of the motion.

Initially swinging from the background, the pendulum is hardly seen to move, creating anticipation.

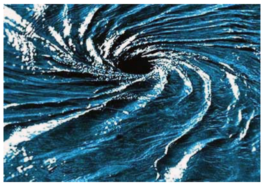
After passing the nadir (lowest point), the pendulum quickly enters the foreground.

The timing near the end is similar to that of a ball thrown straight upward approaching its apex.





Spirals and Sinusoids

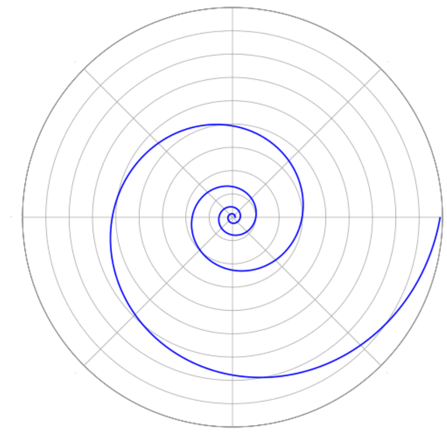


Objects moving in a spiral often go faster as the radius gets smaller, such as water swirling into a drain.

Even if the speed stays constant, as with a tether ball, it takes less and less time to go around since the radius of the circle gets smaller.

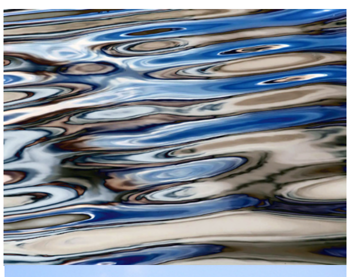
► Spirals and sinusoids are variations of the circular arc.

A spiral is a circular arc with a radius that changes as we turn around in angle.



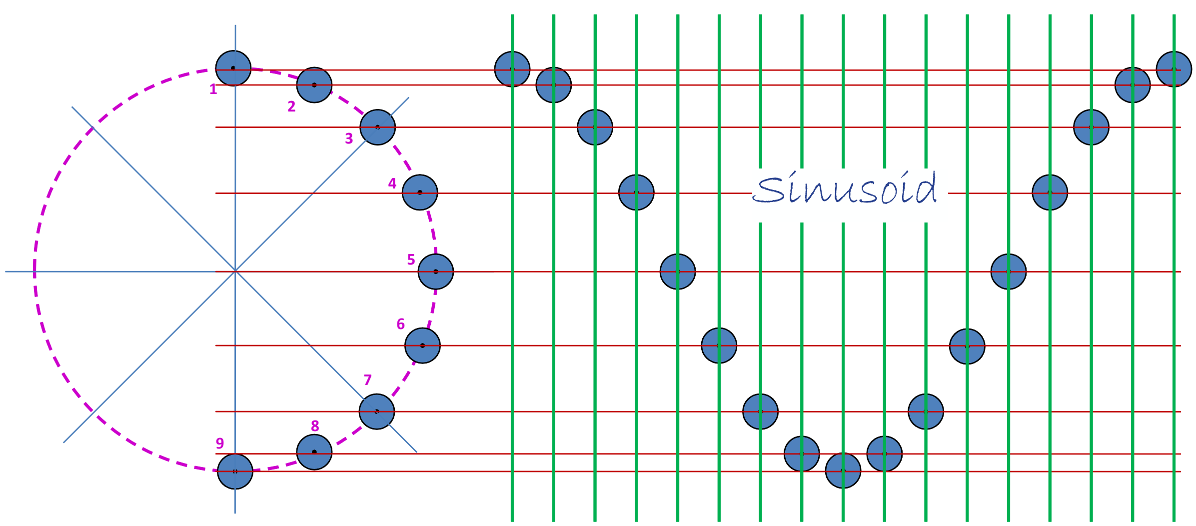
There are different types of spirals, depending on how the radius changes with angle.

This logarithmic spiral, for which the radius increases exponentially, is one of the more common spirals in nature.



Sinusoids are very common in nature and most wave motion follows this pattern. Examples include ripples on a water surface and the flapping of a flag.

A sinusoid is created when uniform rotation goes up and down while at the same time a uniform shift occurs to the side.

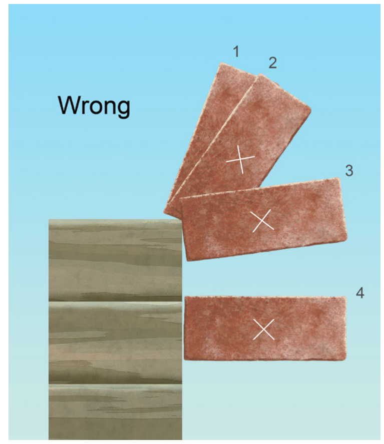


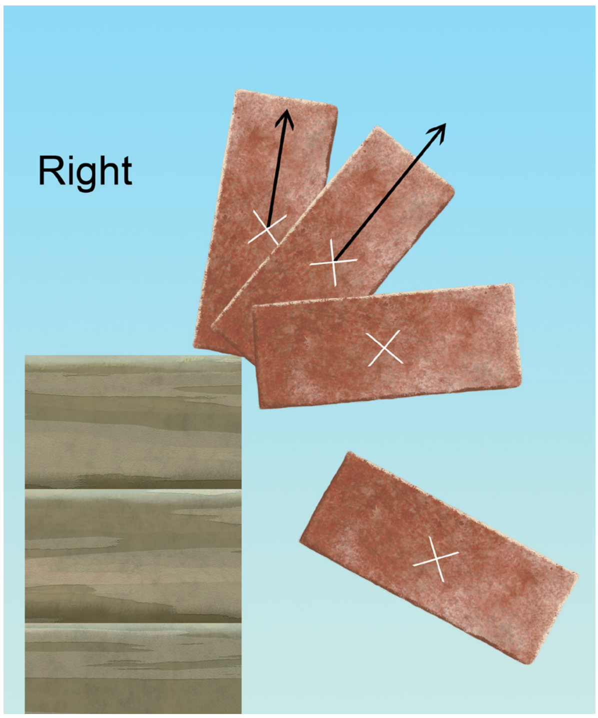
Tipping Forward

► In the brick drop animation test the brick rotates as it tips over and then falls off the table. Due to its rotation the brick acquires a horizontal velocity causing it to move away from the table.

As it picks up speed, the brick acts as if a force is pulling it away from the table. This is known as the centrifugal force.

The brick does not fall this way.



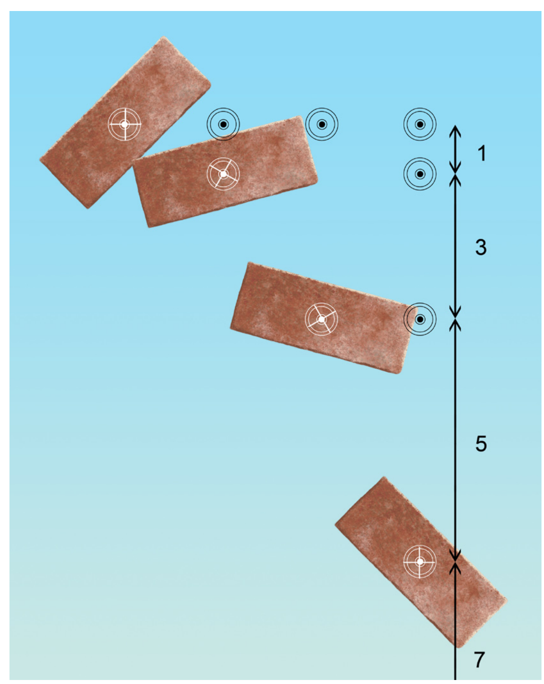


You experience centrifugal force when, as a passenger in a car making a sharp circular turn, you feel yourself pulled outward, away from the center of the circle.

Once the brick slips off the edge of the table the path of action is no longer a circular arc.

In the air, the brick falls in a fashion similar to the falling ball, discussed in the "Physics of Timing and Spacing" tutorial, except that it is not falling straight down.

The brick's falling motion is a combination of vertical falling plus a horizontal motion.



Link: **The origin and nature of centrifugal force is described in the another tutorial (Physics of Creating Action)**

Timing the Brick Drop

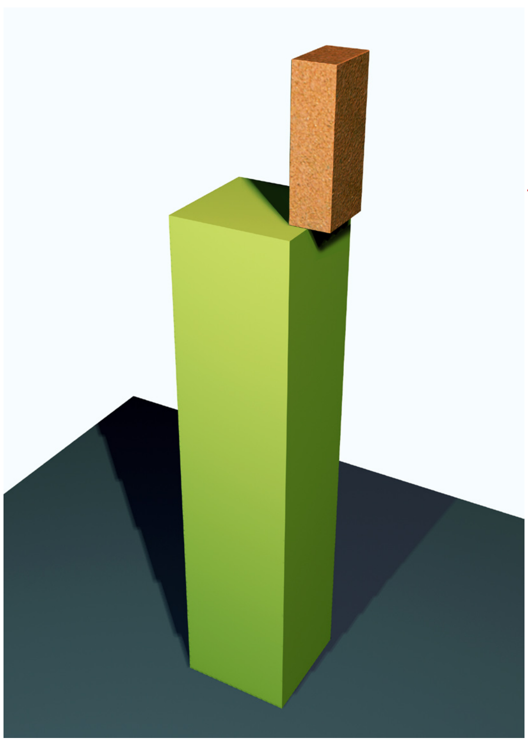
Time (sec.)	Frames	Distance Fallen from Apex
1/24	1	1/3 inch
1/12	2	1 1/3 inches
1/8	3	3 inches
1/6	4	5 1/3 inches
1/4	6	1 foot
1/3	8	1 3/4 feet
1/2	12	4 feet
2/3	16	7 feet
3/4	18	9 feet
1	24	16 feet

Distance Fallen = (1/3 inch) x (Frames) x (Frames)

► The timing of the brick while in the air is the same as that of an object falling straight down, simply adding a uniform horizontal motion to the slowing out in the vertical direction.

The "Distance Fallen table (discussed in the "Physics of Timing and Spacing" tutorial) may be used to approximate the timing of the brick as it falls.

Remember that as long as air resistance isn't significant, all objects fall at the same rate. For a brick, air resistance is negligible unless the brick falls from the top of a tall building.



Question: **After tipping over, about how many frames will this brick be in the air? A standard brick is 2¼ x 3¾ x 8 inches**

Answer: **The height looks to be about 4 brick lengths (which is 32 inches). From the table, that looks to be around 10 frames. From the formula, the distance fallen in 10 frames is 33 1/3 inches, so that's about right.**

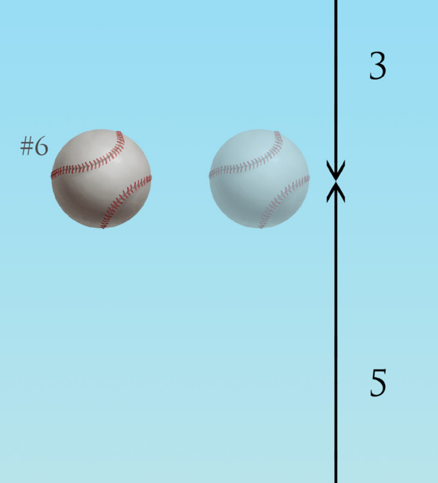
Tip: **The brick drop test is challenging for traditional animators because it is difficult to draw the brick with consistent volume and shape, especially in perspective. This consistency is most important when the brick is not moving, such as at the beginning and the end of the animation.**

Moving & Falling

► The path of action of an object moving horizontally as it falls vertically is a parabolic arc.

A good example of this is a ball rolling off a table.

The parabolic arc is the combination of uniform horizontal motion and accelerating vertical motion. Horizontal distances equally spaced as with uniform motion.

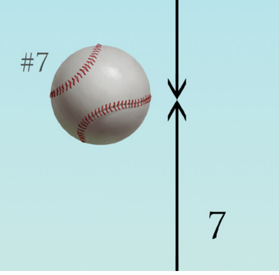


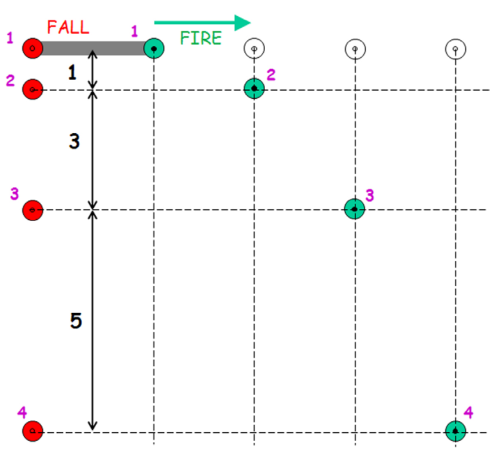
Falling starts with key #4, with vertical distances increasing as 1:3:5:7:… (see the Odd Rule in the "Physics of Timing and Spacing" tutorial).

Note that the distances fallen do not depend on how fast the ball rolls off the table.

Question: **This softball is 4 inches in diameter. How many frames are there between the key drawings, that is, are we shooting on ones, twos, or threes?**

Answer: **The distance between key #4 and #5 is about 3 inches. From the table on the previous page we see that it takes three frames to fall that distance so we're shooting on threes.**





Experiment: **There is a classic physics demonstration in which one ball is released and falls straight down while a second ball is simultaneously fired horizontally.**

As they fall the balls are always at the same height so they hit the ground at the same time. Really!

Try it yourself by placing a flat knife on the corner of a smooth table. Place a coin on the knife tip and a second coin on the table, next to the knife's handle. Quickly slide the knife sideways so the first coin falls and the second is fired sideways. Listen for when they hit the ground.

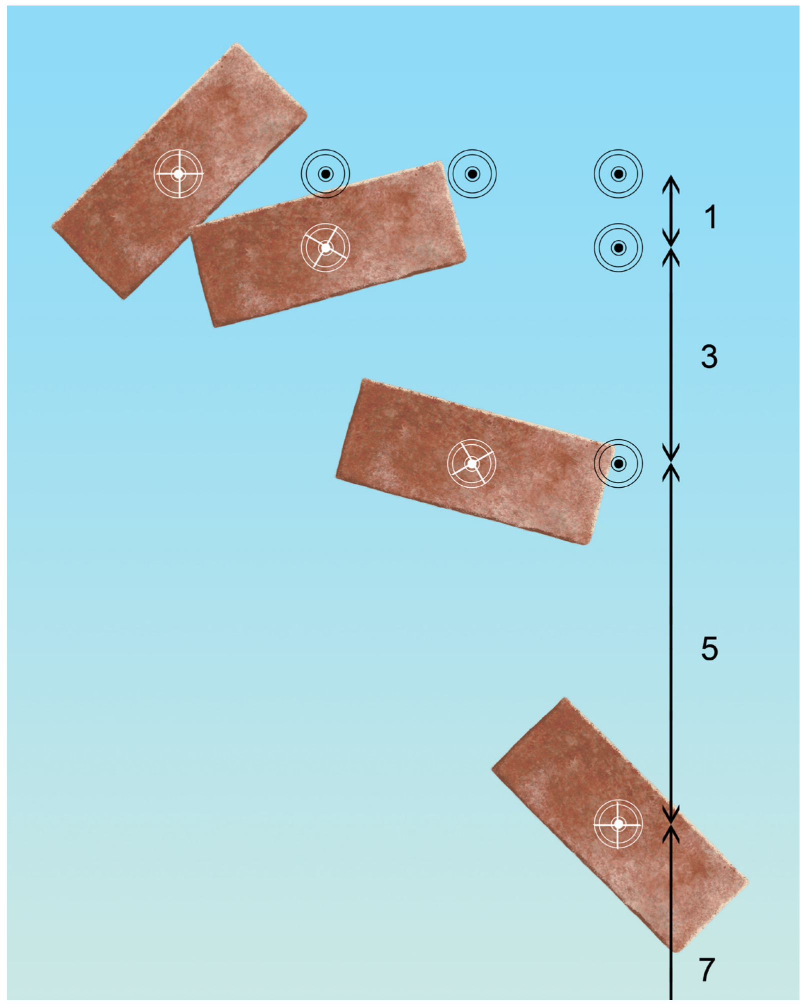
Spinning & Tumbling

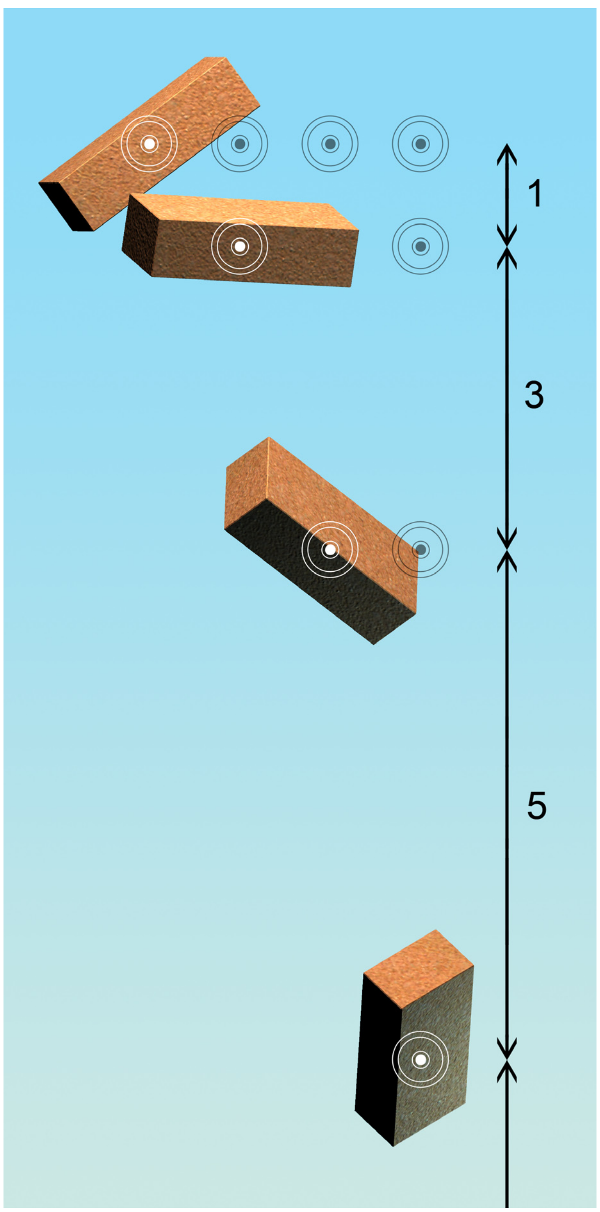
A good example of tumbling is seen when a chair is thrown.

► A falling brick may turn by simple spinning around its center or it may turn by a more complicated tumbling motion.

In simple spinning, the angle rotates at a constant rate.

A brick tipped 45° as it loses contact with the table will fall spinning about 30° every two frames.





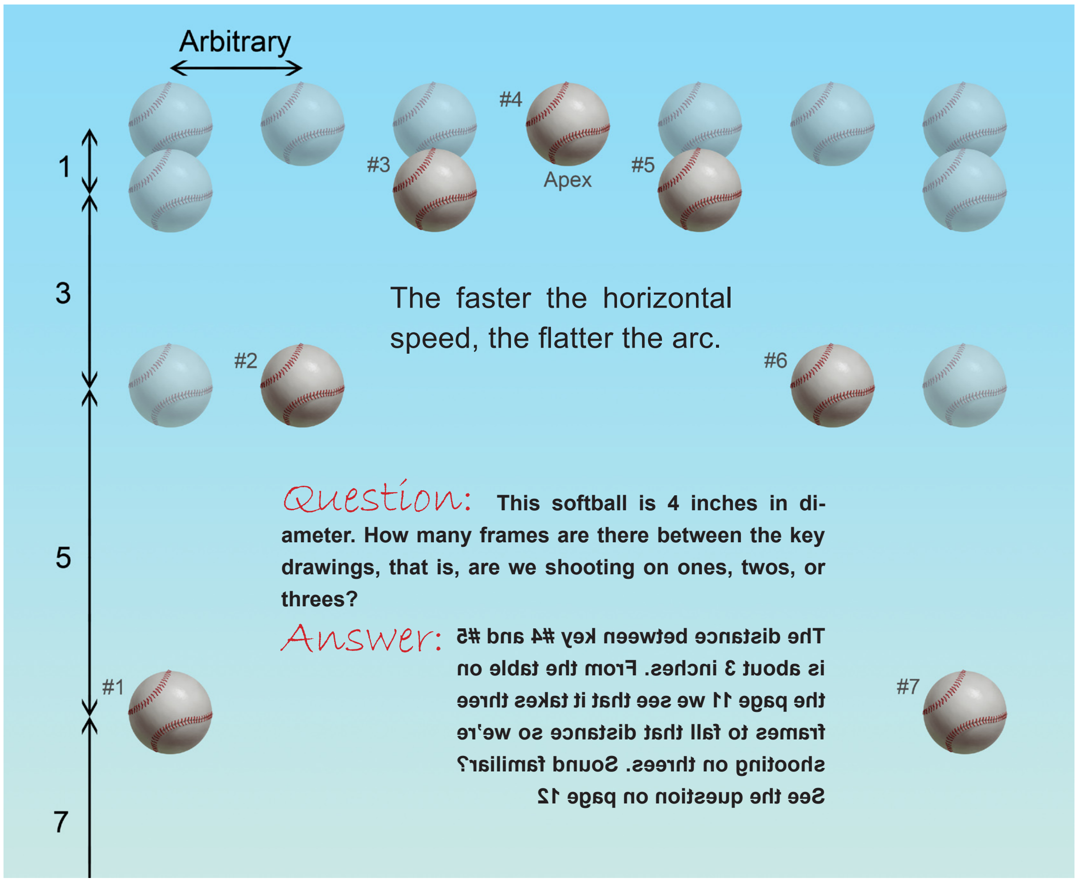
In some cases spinning is unstable and tumbling occurs. This is especially common for irregularly shaped objects and for regular shapes when they're not spinning down the short axis nor down the long axis, such as the brick shown on the left.

There is no simple way to describe tumbling. However, the brick's center still follows the same path of action.

More Parabolic Arcs

► When a ball is thrown upward but not straight up, the path of action is a parabolic arc.

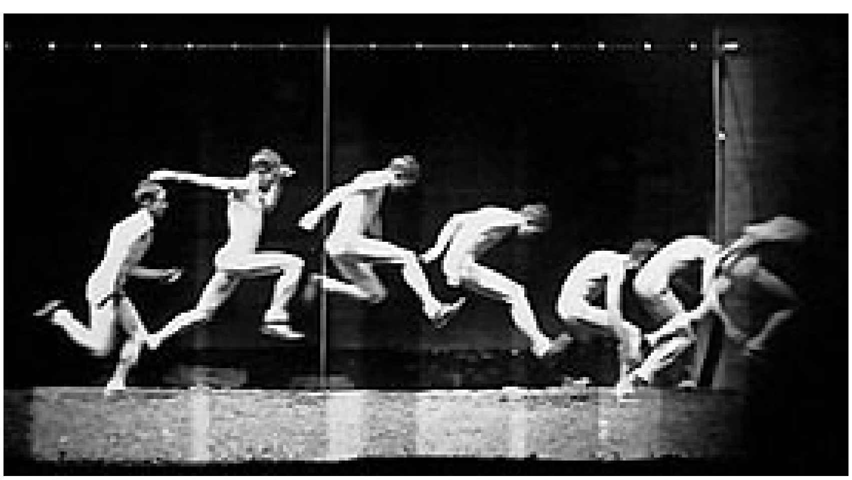
In a parabolic arc the horizontal spacing is constant and uniform while the vertical spacing follows the “Odd Rule” (see “Physics of Timing and Spacing” tutorial).



Question: **This softball is 4 inches in diameter. How many frames are there between the key drawings, that is, are we shooting on ones, twos, or threes?**

Answer: **The distance between key #4 and #5 is about 3 inches. From the table on the page 11 we see that it takes three frames to fall that distance so we're shooting on threes. Sound familiar? See the question on page 12**

The path of action of a jumping character or a steam of water flying through the air is also a parabolic arc.

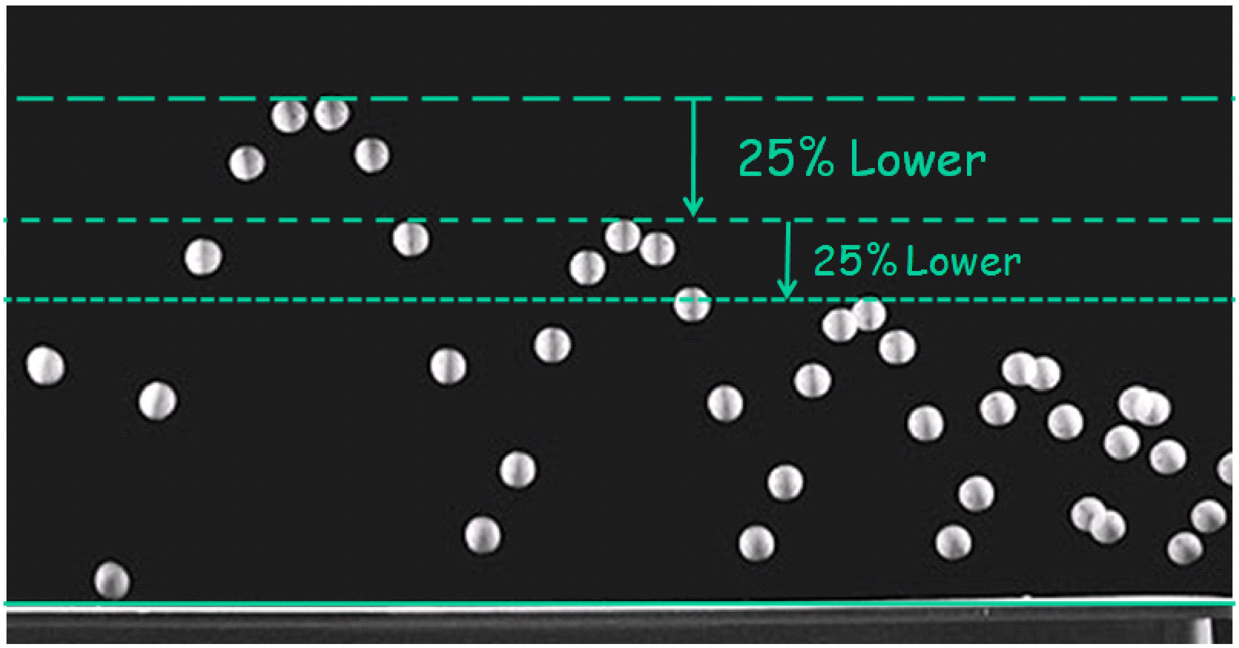


Bouncing

► The path of action of a bouncing ball is a series of parabolic arcs, each with a lower apex (due to some loss of energy on impact with the ground).

Typically, the height of each bounce is lower by the same percentage (about 25% in the picture below).

Bouncing ping pong ball



Bouncing Object	Percent of height lost per bounce
Golf Ball	33%
Billiard Ball	40%
Hand Ball	50%
Tennis Ball	50%
Glass Marble	60%
Steel Ball Bearing	67%
Brick	95%
Clay Ball	100%

To animate a bounce accurately you need to consider the elasticity of both the bouncing object and the surface on which it bounces. For example, a golf ball bouncing on grass loses much more height than bouncing off of concrete.

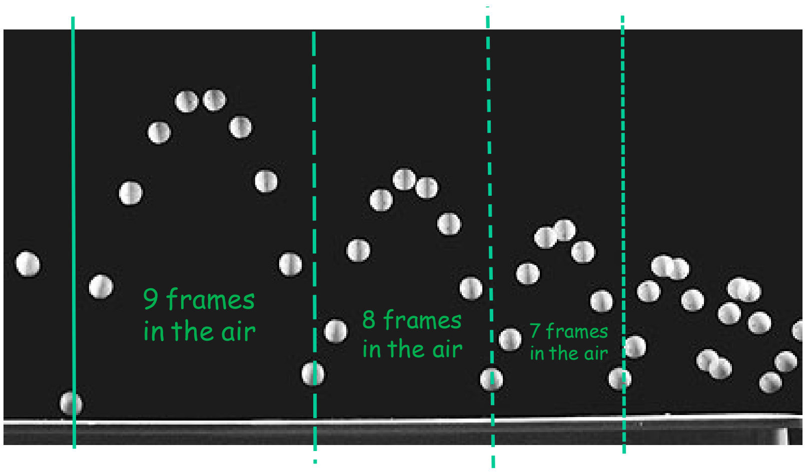
This table shows roughly how much height various objects lose when bouncing on a concrete floor.

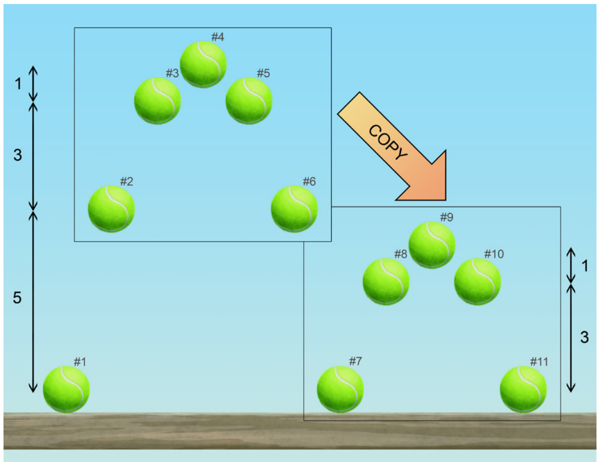
Normally, a bouncing object loses height each time it hits the ground. However, it could gain height if the surface was moving upward on impact (such as bouncing a tennis ball up and down as you move your racket).

Successive Bounces

► Each successive bounce is a similar parabolic arc.

Typically the horizontal spacing stays constant from frame to frame yet the distance traveled with each bounce decreases since the time in the air is also decreasing.



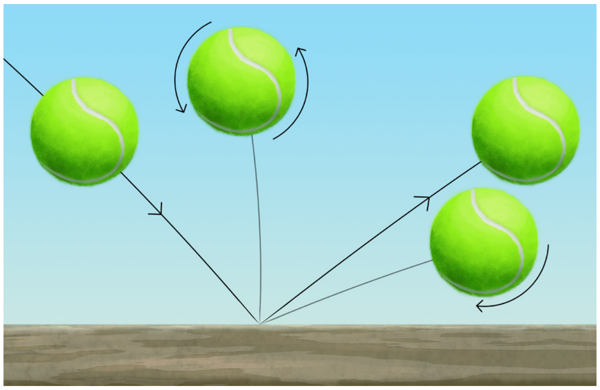


A simple way to do successive bounces is to repeat the upper part of an earlier bounce.

Each successive bounce occurs at a shallower angle, for example if the ball loses half of its height with each bounce and the incoming angle is 45 degrees then the outgoing angle will be 35 degrees.

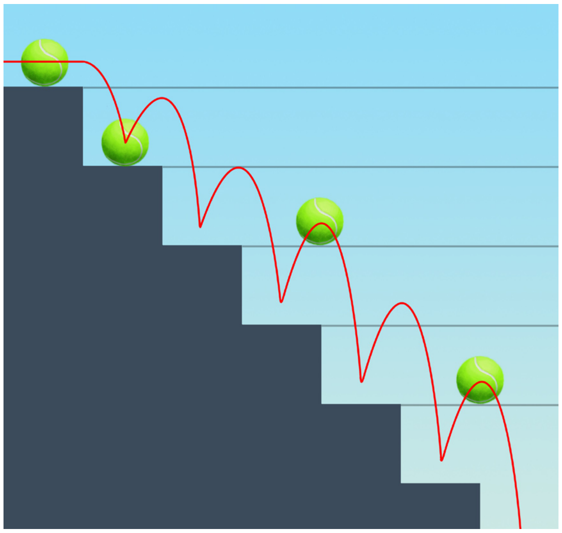
The angle of a bounce will be shallower if the ball has top spin since the spin gives the ball added horizontal velocity on impact. So a ball with top spin will bounce low and fast.

Conversely, a ball with backspin loses horizontal velocity on impact and with enough backspin you can even have the ball come back to your hand. Try it yourself with a SuperBall.



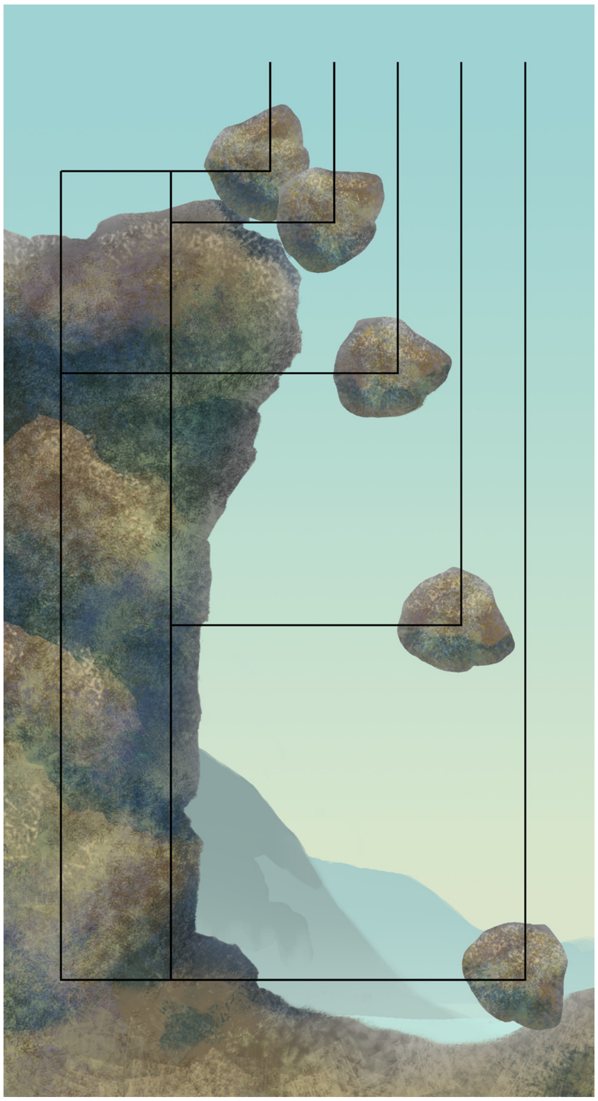
Bouncing down a staircase is an interesting example of successive bounces. As usual, the ball loses a fraction of its energy on each impact but then it picks up speed from falling the extra distance down to the next step.

After the first bounce or two the ball gets into a rhythm and bounces to the same height with each step. For example, if half the height is lost on impact then it bounces up to the height of the previous step, as shown on the right.



Paths of Action and Scale

► The path of action and the spacings on that path are typically not affected by the scale; it's usually the timing of the motion that reveals the scale.



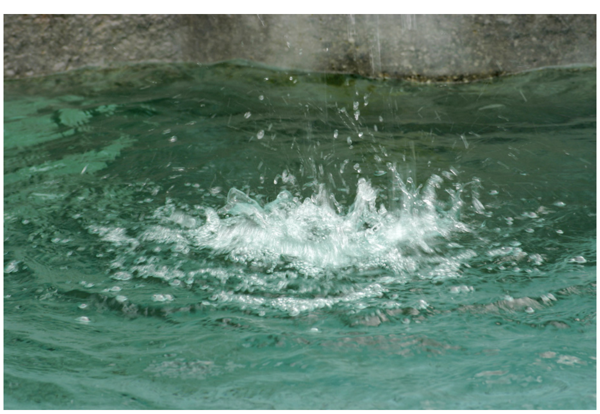
Is this a 6 inch rock falling 4 feet?

Is it a 60 inch boulder falling 40 feet?

The spacings and the path of action are the same in the two cases. However, the timing is different—the rock falls for 12 frames while the boulder falls for about 38 frames.

When scale models are used in films the timing may be corrected using high-speed cameras. For example, if the falling rock is filmed at triple speed (72 frames per second) then it will look like a boulder when played back at normal speed.

Director Ishiro Honda holding upper body of the monster Gojira, also known as Godzilla. (1954)

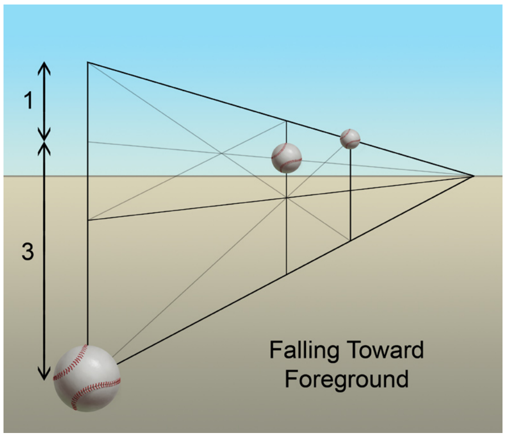


Adjusting for scale is tricky when a variety of forces are at play.

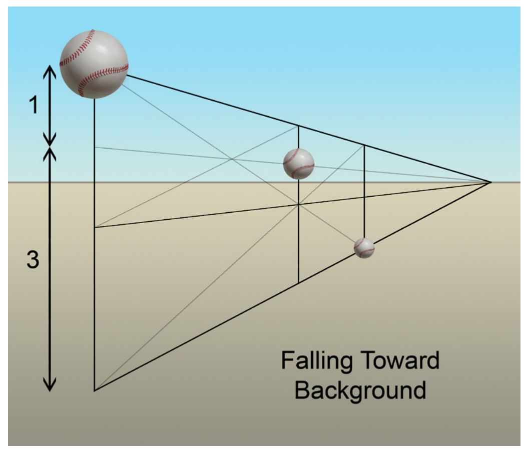
For example, the timing of the droplets splashing through the air depends on gravity but the droplet size depends on surface tension.

Parabolic Arc in Perspective

► To draw a parabolic arc in perspective, such as for a ball moving horizontally as it falls, it's easiest to work using pose-to-pose animation.

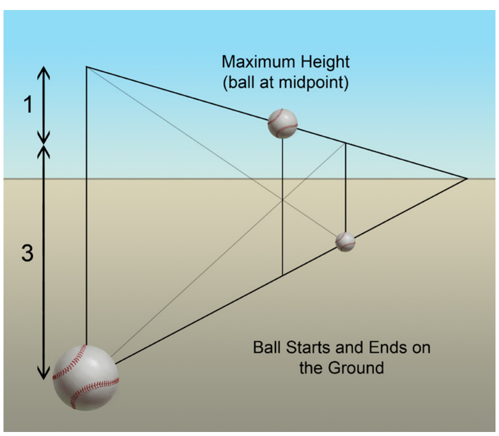


As described in "Physics of Timing and Spacing" tutorial, use the "Fourth Down at Half Time" rule to locate the mid-point between the apex and the lowest key pose.



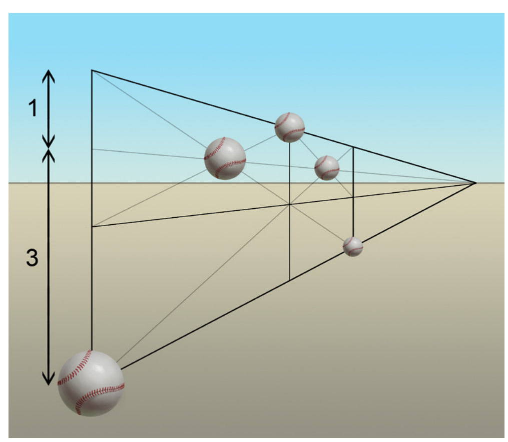
To add more drawings, use the same principles for finding in-betweens as for any falling object.

Remember that slowing out is most noticeable near the apex.



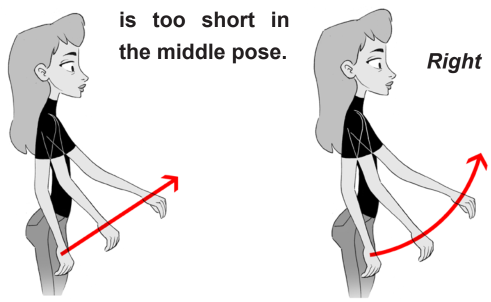
To draw a rising and falling parabolic arc in perspective, select the start and end points on the ground plane.

Decide on the maximum height and draw your perspective lines to the vanishing point.



The intersection of the diagonals locates the midpoint, which gives you the apex of the arc. Now use the "Fourth Down at Half Time" rule to locate the positions on either side of the apex.

Arc Errors



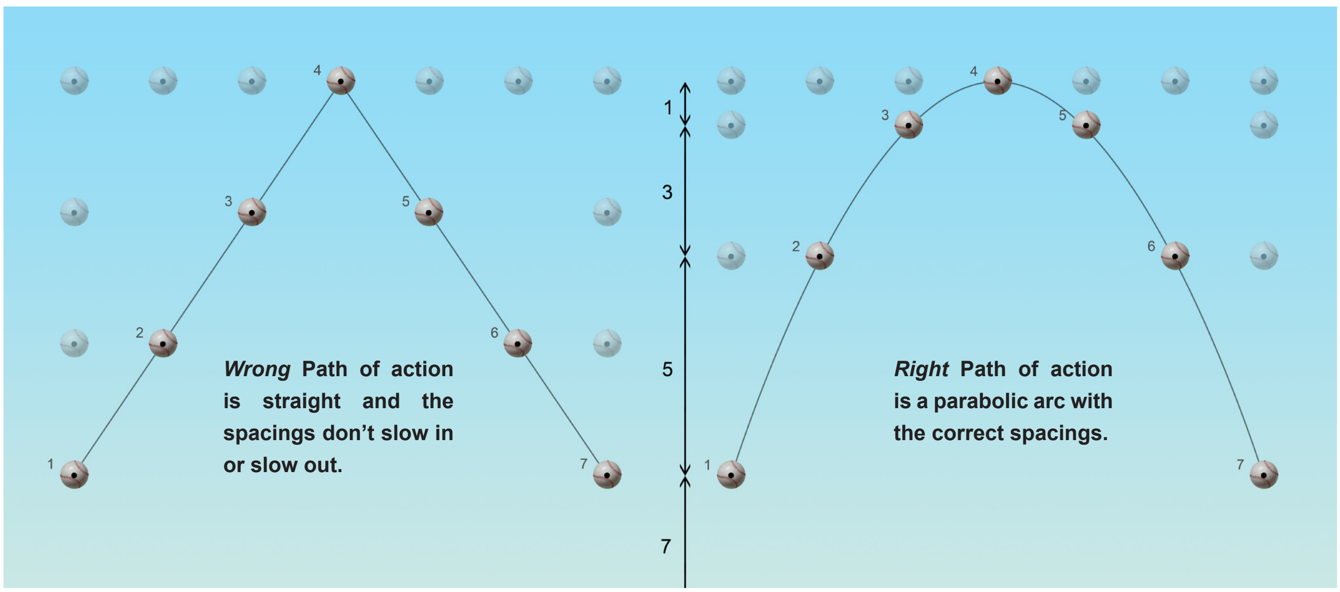
Wrong **The arm is too short in the middle pose.**

Right

► The most common error in drawing arcs is to not draw arcs. In other words, to incorrectly draw a straight path of action.

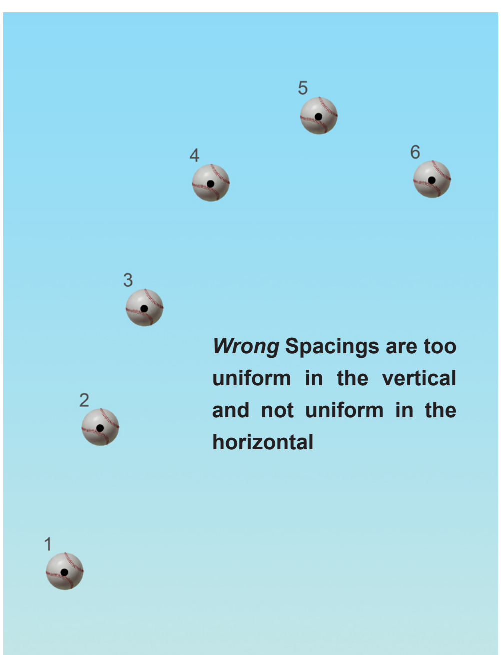
A related error occurs when the timing or the spacing are too uniform.

Other common errors are drawing arcs the wrong shape or with the correct shape but with the wrong spacings (i.e., putting the key drawings at the wrong locations on the arc).



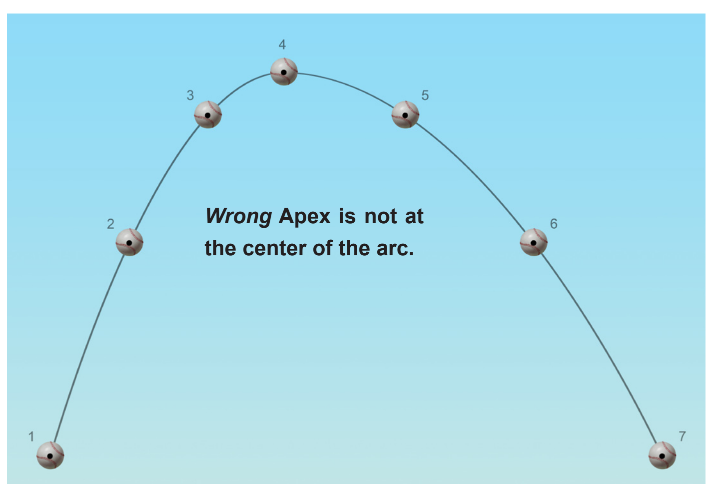
Wrong **Path of action is straight and the spacings don't slow in or slow out.**

Right **Path of action is a parabolic arc with the correct spacings.**



Wrong **Spacings are too uniform in the vertical and not uniform in the horizontal**

Whenever forces act on an object, such as gravity pulling down on the thrown ball, there's an effect on the timing, spacing, and the path of action (and the three are interrelated).



Wrong **Apex is not at the center of the arc.**

Basic Motion Graphs

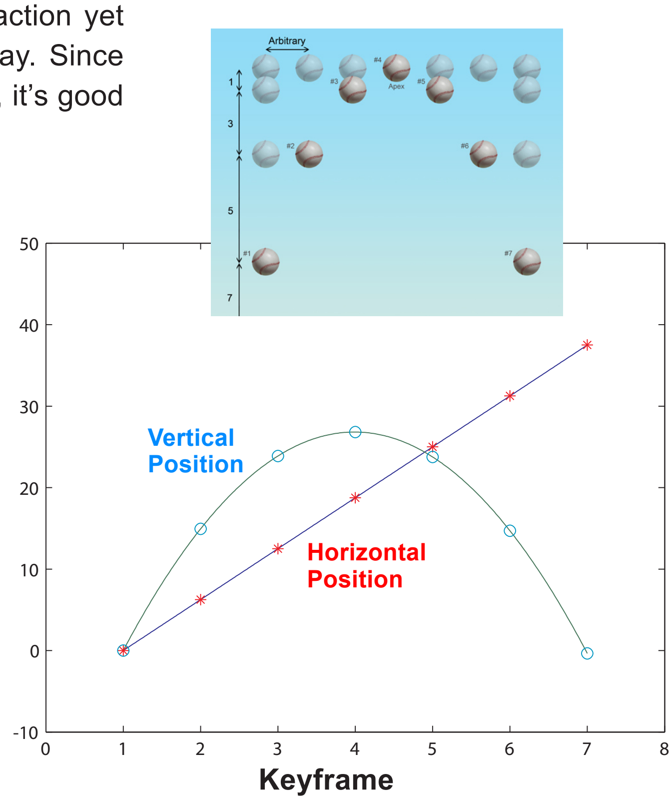
► Motion graphs are closely related to paths of action yet they describe spacing and timing in a different way. Since motion graphs are common in computer animation, it's good to know how they look for different paths of action.

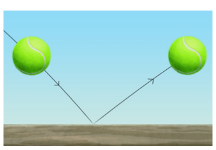
The parabolic path of action of a thrown ball combines constant motion in the horizontal with slowing in and out in the vertical.

Since the horizontal spacings stay uniform the motion graph of the horizontal position is a straight line.

The motion graph for the vertical position is a parabola, just like the path of action.

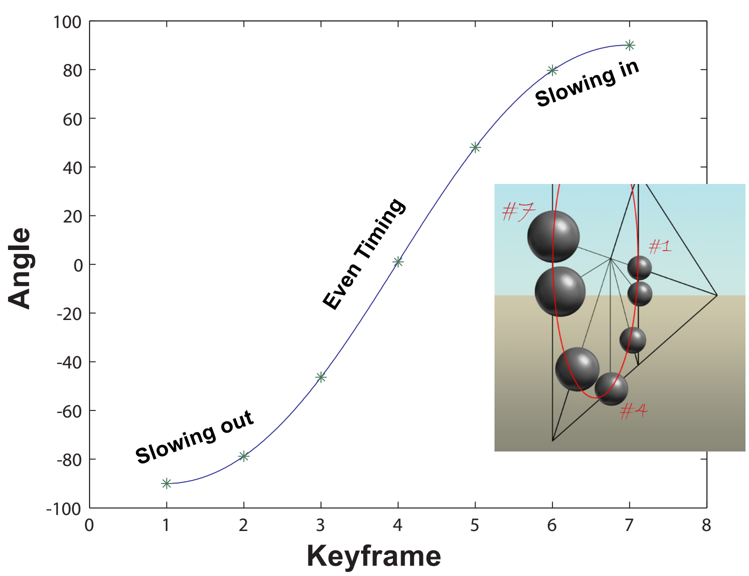
In all cases, the motion graph is steepest at keys where the object is moving the fastest.





Question: **What does the motion graph for a bouncing ball look like?**

Answer: **With no spin on the ball the horizontal speed stays constant so the motion graph for horizontal position is, again, a straight line. The motion graph for the vertical position is a V-shape, in the same shape as the path of action.**



The motion graph for the angle of a swinging pendulum is an S-shaped curve.

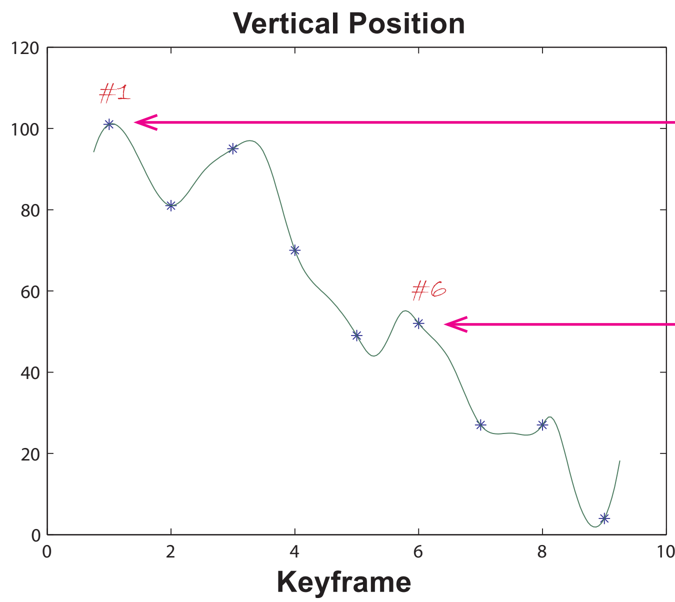
The pendulum slows in and out of each apex; the curve there is not exactly parabolic but close to it.

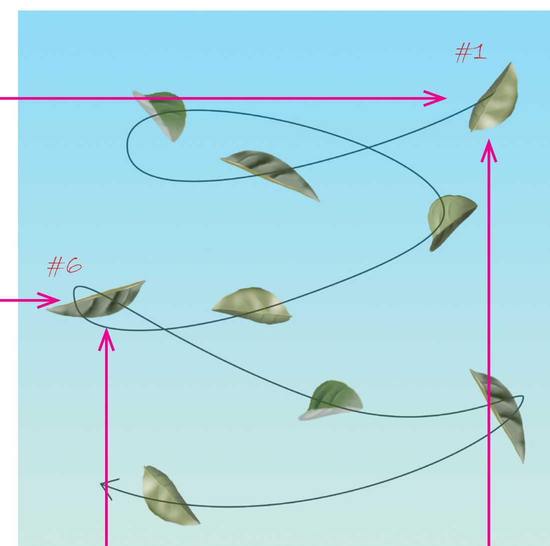
In the middle of the swing, as the pendulum is moving at its highest speed, the motion graph is steep. The graph is nearly a straight line since the speed is nearly constant.

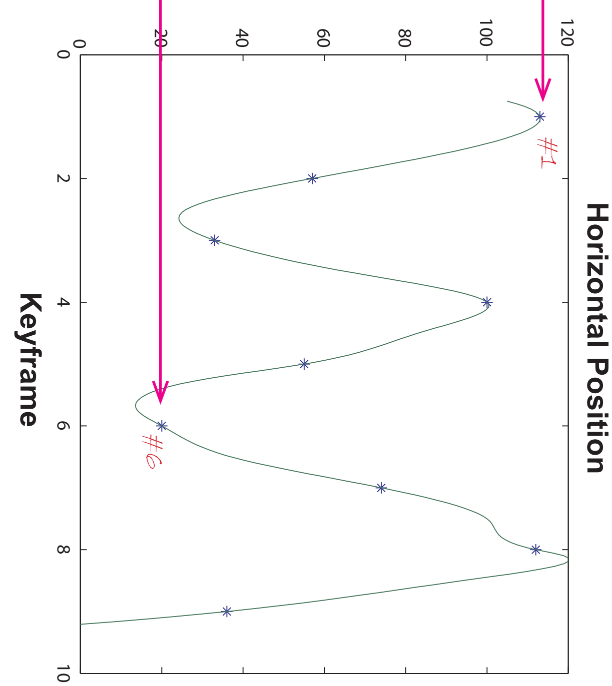
If the pendulum continues swinging back and forth then the S-shaped curve repeats and looks somewhat like a sinusoidal wave.

Complex Motion Graphs

► The motion graph for a complex path of action is irregular and difficult to interpret. Nevertheless, the motion can be understood by breaking it down to examine it piece-by-piece.







The leaf is primarily moving downward yet with each loop it also has a temporary upward surge. Each time that the leaf's motion changes from upward to downward the motion graph of the vertical position has a peak (and there's a valley each time the motion reverses from downward to upward).

As the leaf drifts downward it also goes left to right and then back. This repeated oscillation in the horizontal position is similar to the back and forth rocking of a pendulum.

Notice that, unlike the path of action, the curve on the motion graph can never cross itself; if it did then the object would be in two places at the same time (and the same key frame).

Note: **Sometimes things are easier to see when you change your point of view.**

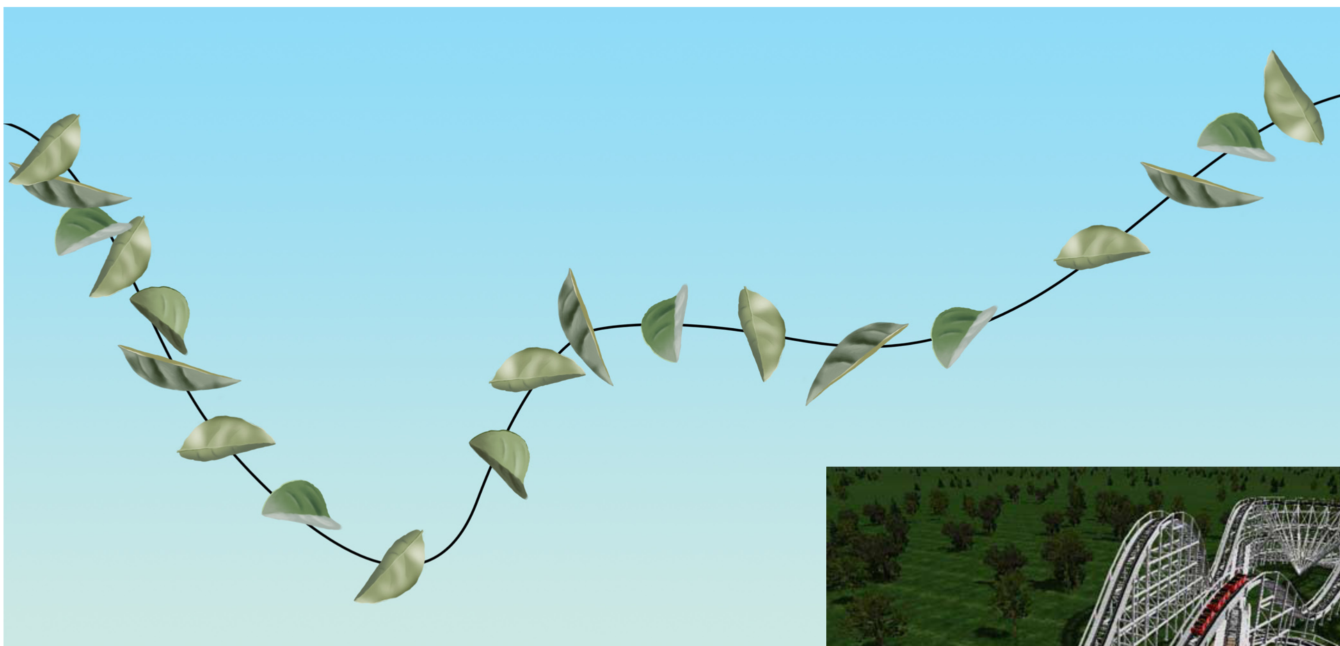
Timing & Spacing for a Path of Action

► Timing and spacing on a complex path of action may be found by breaking it down into segments and using the principles above to estimate the motion on each segment.

Take a leaf floating to the ground. Where the path is flat the leaf slides horizontally with nearly constant speed.

When the path is a steep downward slope the leaf slows out as it slides downward while it slows in on an upward slope.

Rounding a downward curve the timing is like that of a pendulum while on an upward curve it's nearly exponential as the leaf slowly crests over the top of the path.



Picture yourself riding on the moving leaf as if you were on a roller coaster. If your timing and spacing are realistic then you should feel the motion (and so should the audience).

The motion will not follow these timing and spacing patterns if another independent force, such as a blowing wind, pushes on the leaf.

Character Design

► The first two tutorials discussed the timing, spacing, and paths of action for simple objects, like a bouncing ball or a falling brick.

In the next tutorial we'll see how physics can help you with character design, in particular creating balanced poses and believable action poses.

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